Credit Ratings and Security Design*

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PRELIMINARY

Abstract

The poor performance of credit ratings on structured finance products leading up to the financial crisis has prompted investigation into the role of Credit Rating Agencies (CRAs) in designing and marketing these products. We analyze a two-period model where a credit rating agency with reputational concerns both designs and rates structured products that are sold to different clienteles: unconstrained investors and investors with minimum quality requirements. The motivation for pooling assets derives from the attempt to tailor products for constrained investors and reputational incentives. When quality requirements for constrained investors are higher, the CRA's profits decrease and rating inflation increases. If the CRA is considered more likely to be truthful, it can sell two tranches of securities, with ratings inflation only for the top tranche. However, current incentives for inflating ratings will go down, as it prefers to build reputation to capitalize on the prior in the second period.

Keywords: Credit rating agencies, security design, reputation, structured finance

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1 Introduction

The recent financial crisis has prompted much investigation into the role of credit-rating agencies (CRAs). With the dramatic increase in the use of structured finance products, the agencies quickly expanded their business and earned outsize profits (Moody's, for example, tripled its profits between 2002 and 2006). Ratings quality seems to have suffered, as structured finance products were increasingly given top ratings shortly before the financial markets collapsed. In this paper, we ask how the design of such products is influenced by CRAs, and how security design changes with market incentives.

The design of structured finance products is marked by close collaboration between issuers and rating agencies. Issuers depend on rating agencies to certify quality and to be able to sell to regulated investors. Beyond directly paying CRAs for ratings (the "issuer pay" system), Griffin and Tang (2012) write that "The CRA and underwriter may engage in discussion and iteration over assumptions made in the valuation process." Agencies also provide their models to issuers even before the negotiations take place (Benmelech and Dlugosz, 2009). These products are characterized by careful selection of the underlying asset pool and private information about asset quality.

We present a reputation based two period model of rating structured products. Each period an issuer has a set of good and bad assets that it can pool, tranche, and issue securities against. The CRA is long-lived and may be of two types, truthful or opportunistic. Reputation for the CRA consists of the probability that investors perceive it to be truthful. This perception can change according to inferences from ratings and security performance. There are two types of rational investors[1], constrained and unconstrained. Constrained investors need the quality of securities to be above a certain level, while unconstrained investors can purchase any type of security.

We provide two new motivations for the pooling of assets. First, in our model structuring motives derive from the need to tailor products for constrained investors. Second, a CRA can balance the informational advantage over investors with the need to maintain its reputation by choosing the right mix of good and bad assets to include. We also find that when quality requirements for constrained investors are higher, the CRA's profits decrease as it is harder to place assets. Rating inflation increases in response, as lower future profits imply more incentives to take advantage of current investors.

Reputational incentives affect the equilibrium configuration and design of tranches. If the CRA is considered more likely to be truthful, it can sell two

[1]By rational, we mean that they make inferences based on available information using Bayes' rule when possible and maximize their payoff given their constraints.

tranches of securities, with rating inflation only for the top tranche. As its reputation decreases, it may only sell one tranche that has inflated ratings (to unconstrained investors) or not be hired by the issuer. However, for a CRA who is perceived to be more truthful, current incentives for inflating ratings will go down, as it prefers to build reputation to capitalize on the prior in the second period. A larger payoff from retaining assets can also make the CRA more willing to inflate ratings.

The key building blocks of our model are as follows:

- *Security design*: The assets can be pooled, tranched, and/or retained by the issuer. The design is constrained by the incentive to pass off bad securities as good ones (a lemons problem) and the demands of investors, but buffeted by the possibility of milking reputation.

- *Reputation concerns for CRAs*: As rating agencies executives often argue, CRAs are concerned about maintaining their reputation for providing timely and accurate assessments of default risk.

- *Clientele effects*: A principal motivation for securitization is to apportion risk to investor groups with heterogeneous preferences for risk. The obvious example of this was the increased demand for safe investments in the 2000s by regulated entities (e.g. banks, pension funds, insurance companies).

There is substantial evidence of asymmetric information and strategic asset pool selection for structured finance products. Downing, Jaffee, and Wallace (2009) compare the performance of pools of mortgages that are pass-through MBS with no tranching with securitized REMICs (Real Estate Mortgage Investment Conduits) with tranching. The extra layer of securitization and anonymity in sales allows for a selection of worse performing pools due to private information. This is shown to be true with ex-post performance data. Moreover, there is a "lemons spread" due to rational discounting of these securities. An, Deng, and Gabriel (2011) show that portfolio lenders use private information to pass off lower quality loans to commercial mortgage backed securities (CMBS). Conduit lenders, who originate loans for direct sale into securitization markets do not select loans and hence have higher quality loans conditioning on the observables. The analysis shows a lemons discount for portfolio loans. This lemons discount is lower for multifamily loans, which have lower levels of uncertainty and lender private information than retail, office, and industrial loans. Elul (2011) demonstrates that securitized mortgages perform worse than portfolio loans, with the largest differences in prime mortgages in private (non-GSE) securitizations, consistent with the presence of adverse selection. Ashcraft, Goldsmith-Pinkham, and Vickery (2011) find that the MBS deals that were most likely to underperform were the ones with more interest-only loans (because of limited performance history) and lower documentation, that is, loans that were more opaque or difficult to evaluate.

We find that rating inflation occurs for the highest quality tranche levels. In the data, Gorton and Metrick (2012) show that AAA-rated asset backed securities have significantly higher cumulative default rates compared to AAA-rated corporate bonds. This is also true for lower rating categories, but the differences lessen as ratings worsen. Ashcraft, Goldsmith-Pinkham, and Vickery (2011) find that as MBS issuance volume shot up between 2005 and mid-2007, ratings quality declined. Specifically, subordination levels[2] for subprime and Alt-A MBS deals decreased over this period when conditioning on the overall risk of the deal. Subsequent ratings downgrades for the 2005 to mid-2007 cohorts were dramatically larger than for previous cohorts. Griffin and Tang (2012) show that CRA adjustments to their models' predictions of credit risk in the CDO market were positively related to future downgrades. These adjustments were overwhelmingly positive and the amount adjusted (the width of the AAA tranche) increased sharply from 2003 to 2007 (from 6% to 18.2%). He, Qian, and Strahan (2012) find that top rated MBS tranches sold by larger issuers[3] performed significantly worse (prices drop more) and have higher initial yields than those sold by small issuers during the boom period of 2004 to 2006. Stanton and Wallace (2012) demonstrate that the spread between CMBS and corporate bond yields for ratings AA and AAA fell significantly after 2002 (and did not fall for bonds with worse ratings), when risk-based capital requirements for top rated CMBS were lowered significantly. Also, CMBS rated below AA were upgraded to AA or AAA significantly more than the rate observed in a comparable sample of RMBS leading up to the crisis.

In the following subsection, we review related theoretical work. In Section 2, we examine the problem of the issuer when there is no rating agency. In Section 3, we add a CRA and analyze the resulting allocation in a static framework. In Section 4, we look at a 2 period reputation game. Section 5 concludes. All proofs are in the Appendix.

[2]The subordination level they use is the fraction of the deal that is junior to the AAA tranche. A smaller fraction means that the AAA tranche is less "protected" from defaults, and therefore less costly from the issuer's point of view.

[3]They define larger by market share in terms of deals. As a robustness check, they also look at market share in terms of dollars and find similar results.

1.1 Theoretical Literature

The research which is closest to this paper are work by DeMarzo and Duffie (1999), DeMarzo (2005), and Hartman-Glaser (2012). In DeMarzo and Duffie (1999), an issuer can first apportion some of the cash flows of a project into a security before it has private information about the project (designing the security) and then after learning private information about the project can retain a fraction of the security and sell off the rest. The second stage is a signaling problem where the issuer faces the tradeoff between wanting to sell its entire position and use the cash for other projects, or retaining some fraction, which signals higher value. For some security designs the optimal design is debt. This differs from our paper in that here the issuer (via the CRA) has another tool beyond retention to sell, which is reputation. Even in the one period problem, the presence of an honest CRA makes sale possible. CRAs make reputation possible as most issuers do not have a long observable issuance performance history. We also have a clientele effect. DeMarzo (2005) uses a variant of the model of DeMarzo and Duffie (1999) to demonstrate two effects of pooling assets. The information destruction effect occurs because the issuer loses his private information advantage over each asset if they are pooled. The risk diversification effect allows the issuer to create low risk securities. In our model pooling occurs to cater to constrained investors, and the lemons effect is weaker due to reputational forces. Hartman-Glaser (2012) has a long lived issuer who can be truthful or opportunistic. The issuer signals through amount retained, an explicitly costly signal and a type of "security design" (although restrictive). In our paper, we focus on the ability of the issuer to select assets and pooling and tranching can occur due to the clientele effect.

In addition to their empirical results, An, Deng, and Gabriel (2011) have a theoretical model where a portfolio lender can only pass off some loans because of the lemons problem and must sell at a discount. Their results suggest that the magnitude of the lemons discount associated with portfolio loan sales varies positively with the dispersion of loan quality in the pool and inversely with the seller's cost of holding the loans in its portfolio.

There is a literature on security design and information acquisition by investors initiated by Myers and Majluf (1984) that looks at how securities can be split between information sensitive and information insensitive parts (Boot and Thakor (1993), Fulghieri and Lukin (2001), Dang, Gorton, and Holmstrom (2011), and Hanson and Sundaram (2012)). We do not allow for differential information among investors, but include heterogeneity in demand as well as a rating agency whose structuring incentives are reputational.

Mathis, McAndrews, and Rochet (2009) examines how a CRA's concern for its reputation affects its ratings quality. They present a dynamic model of reputation in which a monopolist CRA may mix between lying and truthtelling to build up/exploit its reputation. The authors focus on whether an equilibrium in which the CRA tells the truth in every period exists, and they demonstrate that truthtelling incentives are weaker when the CRA has more business from rating complex products. Strausz (2005) is similar in structure to Mathis et al. (2009), but examines information intermediaries in general. Bar-Isaac and Shapiro (2012) incorporate economic shocks and show that CRA accuracy may be countercyclical, which is also consistent with our results.

In addition to Mathis et al. (2009), there are several other recent theoretical papers on CRAs. Fulghieri, Strobl and Xia (2011) focus on the effect of unsolicited ratings on CRA and issuer incentives. Bolton, Freixas, and Shapiro (2012) demonstrate that competition among CRAs may reduce welfare due to shopping by issuers. Conflicts of interest for CRAs may be higher when exogenous reputation costs are lower and there are more naïve investors. Skreta and Veldkamp (2009) and Sangiorgi, Sokobin, and Spatt (2009) assume that CRAs relay their information truthfully, and they demonstrate how noisier information creates more opportunity for issuers to take advantage of a naive clientele through shopping. In Pagano and Volpin (2012), CRAs also have no conflicts of interest, but can choose ratings to be more or less opaque depending on what the issuer asks for. They show that opacity can enhance liquidity in the primary market but may cause a market freeze in the secondary market.

In the economics literature, Albano and Lizzeri (2001) extends the framework of Lizzeri (1999) and has a producer that can choose a quality of a good that is unobservable to consumers but observable to a certification intermediary. The intermediary commits to a fee schedule and a disclosure rule. The optimal allocation involves underprovision of quality. Our paper differs in several ways. Rather than commit to a disclosure rule, the rating agency in our model uses reputation as a disciplining device. We also have heterogeneous investors.

2 The Model without a Rating Agency

We begin with two types of agents: an issuer and investors. All agents are risk neutral. We will analyze the issuers' problem first without any rating agency, and then look at the effect of introducing a ratings agency.

The issuer has assets of measure $M \geq 2$.[4] A mass μ of the assets are good

[4] The rationale for this inequality will be explained below.

and are worth G to investors. A mass $M - \mu$ are bad and are worth B to investors. The issuer's valuations of the assets are lower than the investors' values for the assets: a good asset is worth g and a bad asset is worth b to the issuer. This can occur for several reasons: the issuer has valuable alternative investment opportunities, has regulatory capital requirements for holding the assets, and/or the need to transfer risk off of its balance sheet.

We assume the following ordering:

$$b < B < g < G$$

The inequality of $g > B$ indicates that issuers prefer to keep good assets rather than sell them off if investors perceive them as B.

There are a measure 1 of atomistic investors each willing to buy 1 security. Investors can be one of two types: constrained (measure α) and unconstrained (measure $1 - \alpha$). Constrained investors will only purchase securities that they believe are high quality and have value of at least $\bar{V}$. We assume that $B < \bar{V} < G$, which implies that constrained investors would not purchase a security that has a payoff of B.

Constrained investors may be constrained because of regulations (for example banks, pension funds, and insurance companies are often restricted in the types of assets they may hold) or because of their portfolio hedging requirements. In practice, regulations currently require these types of institutions to hold investment products that have specific ratings. We relax this requirement for two reasons. First, regulations are being changed to weaken the dependence on ratings, and are tending toward using institutional risk models. Second, we do not want ratings to be driven by regulation, which has been discussed amply in the literature (see White (2010), and Opp, Opp, and Harris (2012)). An important argument for securitization is the clientele effect, which is what we are directly modeling here.

The unconstrained investors are willing to purchase any security. They may be hedge funds or other institutional investors. We assume both types of investor are rational in the sense that they update given available information and maximize their payoff.[5]

The issuer can put together portfolios of good and bad assets through *securitization*. We define securitization as selling securities based on the payoffs of the portfolio. We restrict the space of securities by defining the

[5] There has been much discussion about the naivete of investors in the RMBS market, e.g. see Bolton, Freixas, and Shapiro (2012). However, not all structured finance markets are necessarily characterized in such a way, as Stanton and Wallace (2012) point out: "All agents in the CMBS market can reasonably be viewed as sophisticated, informed investors."

payoff of a security as the average payoff of an underlying pool of assets. The average payoff for securities based on portfolio i will be defined by the fraction of assets in the portfolio that are good γ_i. The actual average payoff, or valuation, will be represented by V_i.

In this model, the maximum number of different types of securities it could create are two: one for unconstrained investors (U), one for constrained investors (C). We will call a block of securities designated for a specific type of investor a tranche. The assets retained by the issuer (I) may be considered the equity tranche. The quantity of a specific tranche is denoted by q_i, where $i \in \{U, C, I\}$. Securitization changes the quality profile, but does not change the overall quality of the assets. The constraints on securitization are:

$$q_U\gamma_U + q_C\gamma_C + q_I\gamma_I = \mu, \tag{1}$$

$$q_U(1-\gamma_U) + q_C(1-\gamma_C) + q_I(1-\gamma_I) = M - \mu. \tag{2}$$

The first equation says that the sum of the claims on good assets equals the amount of good assets. The second equation is analogous for bad assets. These equations imply that $q_I + q_U + q_C = M$. This, of course, is an extremely stylized model of how securitization works, in practice things are much more complicated (see Coval, Jurek, and Stafford (2009) for a detailed description of the process).

We will assume that the quality demanded by constrained investors is larger than the amount of high quality investments that can be generated:

$$\bar{V} > (\mu G + (\alpha - \mu)B)/\alpha \tag{A1}$$

The right hand side of this equation describes the maximum value of a security that can be sold to all constrained investors. This value is attained by allocating all good assets to the constrained tranche. The inequality also implies that $\alpha > \mu$. This essentially ensures that the issuer can't create enough good securities to satisfy the demand of all constrained investors, which lies at the heart of the asymmetric information problem.

We also assume that the issuer can't observe investor types. This will not matter, as the issuer can use simple incentive contracts (giving an epsilon more of surplus to unconstrained investors) to perfectly screen them.

Issuers make take it or leave it offers to investors. The reservation utility of all investors is normalized to zero.

2.1 Full Information

Suppose that there is full information about the securities' profile. The issuer has two choices. It can sell just to unconstrained types, in which case its payoff net of the initial value of the portfolio, $(M-\mu)\,b+\mu g$, is

$$q_U\left(\gamma_U(G-g)+(1-\gamma_U)(B-b)\right). \tag{3}$$

The second possibility for the issuer is to sell to both types of investors. In this case, the net payoff is

$$q'_U(\gamma'_U(G-g)+(1-\gamma'_U)(B-b))+q'_C(\gamma'_C\,(G-g)+(1-\gamma'_C)\,(B-b)). \tag{4}$$

Here, the issuer tries to sell as much as it can to the constrained investors, and passes the rest off to the unconstrained investors. Notice that just selling to constrained investors (and not to unconstrained investors) is dominated by this option, as unconstrained investors place a higher value on any remaining tranche than the issuer does. We now solve for the issuer's optimal allocation.

Lemma 1 *Given A1, the optimal allocation consists of:*

1. *The pool of assets for the constrained investors: the issuer places all of the good assets and an amount of bad assets such that the average value in the pool equals $\bar{V}$.*

2. *The pool of assets for the unconstrained investors: the issuer places only bad assets.*

3. *The remainder is retained by the issuer.*

The issuer's optimal strategy is to sell to as many constrained investors as possible. Since demand from such investors exceeds the potential supply, they are rationed. The remaining securities are set to the lowest quality and split among the unconstrained investors and the issuer.

The issuer therefore engages in securitization when there is full information. It will also retain part of the riskiest securities if necessary to maximize its sales. Notice that the issuer is maximizing total welfare. This will not be the case when there is asymmetric information with a rating agency.

2.2 Asymmetric Information

When the issuer's type is private information, the issuer faces the problem of persuading investors that the securities are of a certain quality. We will demonstrate that this directly leads to a lemons problem. This is similar to the adverse selection problem found in the empirical work of Downing, Jaffee, and Wallace (2009) and An, Deng, and Gabriel (2011), who document a lemons spread and worse ex-post performance when issuers have more scope for selecting the loans that are securitized.

We assume the issuer will offer a range of securities to investors with labels of their quality. Investors will observe the quantity of securities q_i and the reported value of the securities R_i, where $i \in \{U, C\}$. We employ the equilibrium concept of Perfect Bayesian Equilibrium. In the following lemma, we describe the equilibrium allocation.

Lemma 2 *Given A1, the issuer will sell* $1 - \alpha$ *securities of value* B *to unconstrained investors and retain the rest.*

This represents a breakdown of the market typical for adverse selection problems. The issuer can't include any good assets in equilibrium. If it did, and investors believed the good assets were included and raised their valuations, the issuer would then replace the assets with bad ones to capture the extra rents. This temptation leads to only bad assets being sold.

The welfare loss from asymmetric information is equal to $(G-g)\mu + (B-b)\mu\left(G-\bar{V}\right)/\left(\bar{V}-B\right)$, the loss from being unable to sell a tranche backed by a mass μ of good assets and a mass $\mu\left(G-\bar{V}\right)/\left(\bar{V}-B\right)$ of bad assets to the constrained investors.

3 The Model with a Rating Agency

In this section, we examine whether a rating agency can reduce or eliminate the asymmetric information problem. We also study how ratings interact with the structuring of the investments. We focus on a monopoly rating agency.

The CRA reduces the lemons problem through the reputation it acquires over time. We model two types of rating agency: truthful (T) and opportunistic (O). This follows the approach of Fulghieri, Strobl, and Xia (2012) and Mathis, McAndrews, and Rochet (2009) (who in turn follow the classic approach of modeling reputation of Kreps and Wilson (1984) and Milgrom and Roberts (1984)). The opportunistic CRA will announce the value for each security, but will choose its announcement and the security design on the basis of its incentives. The truthful CRA is a behavioral player and announces truthfully the value of the securities it rates. Despite being behavioral, the truthful CRA may be strategic in our model. It chooses the design of the security sold, while being restricted to report the actual value of its design. This is a departure from the literature, which reduces the opportunistic player to a nonstrategic player.[6] The probability of facing a truthful CRA in the first period is given by the prior, θ_1, which, together with the structure of the game and payoffs, is common knowledge. We also assume the issuer knows the type of the rating agency.[7]

The CRA observes perfectly the assets of the issuer and makes a take-it-or-leave-it offer to the issuer. As part of its services, the CRA designs and rates the securities offered by the issuer for a fee f. While in practice, the issuer will initially design the securities and get feedback from the rating agencies about modifications necessary to achieve certain ratings[8], we incorporate this back and forth into one step for simplicity. If the issuer does not use a rating agency it may issue securities nevertheless. Therefore the issuer can get at least its asymmetric information net payoff of $(1-\alpha)(B-b)$ by not purchasing ratings. We will assume the CRA incurs a positive, but arbitrarily small cost of issuing a rating. Hence, in any equilibrium the CRA is hired if and only if it can create additional surplus.

Denote a message that can be sent by a CRA of type $d \in \{T, O\}$ by $m^d = (R_C^d, R_U^d, q_C^d, q_U^d)$ and the set of such messages by M, where R_i^d is the reported value (rating) of a security intended for an investor of type $i \in \{C, U\}$. The quantity q_i^d represents the amount of securities available for investors of type $i \in \{C, U\}$ from an issuer paired with a CRA of type $d \in \{T, O\}$. This quantity is observable to investors. If the issuer does not issue a security intended for group i, we set $q_i^d = 0$ and $R_i^d = \emptyset$. Denote the true valuation of the securities issued by $v^d = (V_C^d, V_U^d)$. A strategy for a CRA of type d is thus $s = (m^d, v^d, f^d) \in S_d$. If the CRA is truthful, then the strategy space is restricted such that $(R_C^T, R_U^T) \equiv (V_C^T, V_U^T)$.

[6] The only exception we are aware of is Hartman-Glaser (2012) where the truthful issuer can decide how much to retain of a security.

[7] As the issuer knows the quality of its securities, this is the most natural assumption; otherwise, both types of rating agency would be involved in a two-sided signaling game as in Bouvard and Levy (2010) and Bar-Isaac and Deb (2012). Other papers on CRAs do not need to make an assumption about this as the issuer has no choice variable.

[8] See details in Griffin and Tang (2012). Rating agencies also provide their basic model to issuers to communicate further. For example, Benmelech and Dlugosz (2009) write, "The CDO Evaluator software [from S&P, publicly available] enabled issuers to structure their CDOs to achieve the highest possible credit rating at the lowest possible cost... the model provided a sensitivity analysis feature that made it easy for issuers to target the highest possible credit rating at the lowest cost."

Let $\beta : M \to \Delta$ be the belief function of the investors, assigning a probability distribution over the set of CRA types upon observing m, so that $\beta(d|m)$ is the conditional belief that a CRA is of type $d \in \{T, O\}$ given a message m. Let $v_i^{\beta}(m)$ be the expected valuation of investors conditional on message m under the beliefs β if $q_i > 0$, and zero otherwise. Each unconstrained investor makes a bid of $p_U(m) = v_C^{\beta}(m)$. Each constrained investor makes a bid of $p_C(m)$, where $p_C(m) = v_C^{\beta}(m)$ if $v_C^{\beta}(m) \geq \bar{V}$ and $p_C(m) = 0$ otherwise.

While we allow ratings to be continuous, in reality, CRAs use discrete ratings. In principle, ratings correspond to ranges of default probabilities - although CRAs do not publish the ranges corresponding to the ratings. Allowing for ratings from a continuous range in the model has several benefits. First, it does not make us impose an arbitrary scaling and allows us to be general. Second, it allows us to abstract from "rating at the edge", i.e. setting securities to the lowest value of a prescribed range. While this may have been an important phenomenon, rational investors should anticipate this and adjust accordingly, thus undoing its effect. Third, even legislation such as the Dodd-Frank bill has recognized that structured finance ratings are different from corporate bond ratings, meaning that we are effectively allowing the rating agency to set its standards.[9]

Note that our assumption that $M \geq 2$ guarantees that the opportunistic CRA has sufficiently many bad assets to create tranches of equal size to the truthful CRA's, containing only bad assets.

To summarize, the timing of the game with one issuer is as follows:

1. Nature selects the type d of the CRA.
2. The CRA offers the issuer a contract for fee f^d.
3. If the issuer accepts, then the CRA designs the securities, selects tranche sizes, and reports values. Otherwise, the issuer designs the securities, selects tranche sizes and sells the securities himself, without any rating.
4. Investors observe the sizes of tranches and reported values and buy securities at their conditional expected value.

We suppose the game is played twice, and that the issuer is different in each period. At the end of the first period, with some probability that increases in the amount of rating inflation, investors discover the type of the opportunistic CRA.

[9] For a basic metric, Gorton (2012) shows that asset backed securities have significantly higher cumulative default rates compared to equivalently rated corporate bonds.

3.1 The Second Period

In this section, we will analyze the second period, when the type of the CRA has not been revealed in the first period. Since this is the last period, the opportunistic CRA has no reputation concerns. Our first result concerns the securities of the issuer at the opportunistic CRA.

Lemma 3 *In any equilibrium, any security rated by the opportunistic CRA will have a value of B.*

Without reputation concerns, the opportunistic CRA has no incentive to include good assets in the pool of assets to sell since the actual composition is not observable to investors.

We say that an equilibrium is *pooling* if it has the property that both types of CRAs report the same values of all securities and the sizes of all tranches are the same (we will also include any equilibrium where the CRAs are not hired in this category). We call any equilibrium which is not pooling and where at least one CRA is hired, a *separating* equilibrium.

Lemma 4 *There is no separating equilibrium.*

This is an important result in the characterization of the equilibrium. If there were a separating equilibrium, the opportunistic CRA would be recognized and the best it could do is sell bad assets to unconstrained investors. As the issuer could do this without the CRA, the CRA would not be hired given the small fixed cost of operating.

Given this result, if a CRA is hired, only pooling equilibria are possible. The possible pooling equilibria where CRAs are active could have securities sold in one tranche only to unconstrained investors, securities sold in one tranche only to constrained investors, and where securities are sold in two tranches, one meant for each type of investor. All of these possible pooling equilibria exist. However, after we refine the set of equilibria, there will no longer be one where securities are sold in one tranche only to constrained investors.

Given the numerous equilibria that can be supported by a variety of off-the-equilibrium path beliefs, we use the refinement concept of *Undefeated Equilibrium*, introduced by Mailath, Okuno-Fujiwara, and Postlewaite (1993). Placing restrictions on off-the-equilibrium path beliefs using a concept such as the Intuitive Criterion (Cho and Kreps, 1987) has little bite in this environment, whereas the Undefeated Equilibrium concept selects a unique equilibrium for a given set of parameters. We give a brief intuitive discussion of the concept here, and define it formally in the appendix.

The undefeated equilibrium concept is used to select among different Pure Strategy Sequential Equilibria (PSE). In our setting, these are equilibria such that (1) each type of CRA is using a pure strategy and maximizing profits given the investors' bids and the other CRA's strategy, (2) each investor bids his expected value conditional upon observed tranche sizes and reported values, and (3) beliefs are calculated using Bayes' rule for tranche sizes and reported values used with positive probability.

A PSE, E, is said to *defeat* another PSE, E', if: (1) there is a message m sent only in E, (2) the set of types K who send this message are all better off in E than in E' and at least one of them is strictly better off, and (3) under E', the beliefs about some such a type are not a posterior off-the-equilibrium path assuming only types in K send m. A PSE is said to be *undefeated* if the game has no other PSE that defeats it.

The undefeated concept essentially works by checking that no types in one equilibrium are better off in another equilibrium where they choose a different action/message.[10]

We now write three conditions which will help define the parameter space for the unique undefeated equilibrium. Let θ_2 be the posterior at the beginning of period 2 if the type of the CRA has not been revealed in period 1.

$$\theta_2 G + (1-\theta_2)B \geq \bar{V} \tag{C1}$$

$$\left(\bar{V} - b\right)\mu\theta_2(G-B)/\left(\bar{V} - B\right) - \mu(g-b) > 0 \tag{C2}$$

$$\theta_2(G-B) > g - b \tag{C3}$$

The first condition states that if the truthful CRA placed only good assets into the tranche for the constrained investors and the opportunistic CRA put only bad assets into that tranche, the constrained investors would be willing to invest. The second condition states that expected profits for a truthful CRA selling two tranches are strictly positive. The third condition says that the truthful CRA strictly prefers to add one more good asset rather than a bad asset to the asset pool being sold.

We now proceed to find the undefeated equilibrium.

[10]While this works by comparing equilibrium payoffs, Mailath, Okuno-Fujiwara, and Postlewaite (1993) suggest this places more realistic restrictions on off-the-equilibrium path beliefs than other concepts by using beliefs from an actual equilibrium. In the examples they examine, this selects the most reasonable equilibria. This concept is also used in several other papers, including Taylor (1999), Gomes (2000), and Fishman and Hagerty (2003).

Proposition 1 *If and only if C1 and C2 hold, the unique undefeated equilibrium (which we denote E_{**}) has two tranches of the following form:*

1. *The tranche designed by the truthful CRA to constrained investors contains all of the good assets and an amount of bad assets such that the average value in the pool equals $\bar{V}$.*

2. *The tranche designed by the truthful CRA to unconstrained investors contains a measure $1-\alpha$ of bad assets.*

3. *Profits for the truthful CRA are equal to:*

$$\left(\bar{V}-b\right)\mu\theta_2(G-B)/\left(\bar{V}-B\right)-\mu(g-b)$$

4. *Profits for the opportunistic CRA are equal to:*

$$\left(\bar{V}-b\right)\mu\theta_2(G-B)/\left(\bar{V}-B\right)$$

5. *The report for the constrained tranche is $R_C=\left(\bar{V}-(1-\theta_2)B\right)/\theta_2$ and the quantity sold for that tranche is $q_C=\mu\theta_2(G-B)/\left(\bar{V}-B\right)$.*

In the proposition, the unique undefeated equilibrium has two tranches: it sells to both constrained investors and unconstrained investors. The issuer at the truthful CRA puts all of its good assets as well as some bad assets in the tranche for the constrained investors, while the issuer at the opportunistic CRA puts in only bad assets. Both put in only bad assets for the unconstrained tranche. Both tranches are priced according to the rational expectations of investors, meaning they are also dependent on the investors' perception that the CRA is truthful. Note that the issuer at the truthful CRA generates strictly lower revenues than in full information and sells off strictly fewer assets. The opportunistic CRA does strictly better than the truthful CRA as it receives the same price and sells off more bad assets (and retains more good assets). The issuer with an opportunistic CRA offloads more bad assets than if there were asymmetric information with no CRA.

For our next set of parameters, we find a unique one tranche undefeated equilibrium.

Proposition 2 *If and only if C1 does not hold and C3 holds, the unique undefeated equilibrium (which we denote E_*) has one tranche of the following form:*

1. *The truthful CRA issues one tranche of size $1-\alpha$ for the unconstrained investors backed by a measure $\min\{1-\alpha,\mu\}$ of good assets. The rest of the assets are bad.*

2. The profits for a truthful CRA are:

$$\min\{1-\alpha,\mu\}\left(\theta_2(G-B)+b-g\right)$$

3. The profits for an opportunistic CRA are:

$$\min\{1-\alpha,\mu\}\theta_2(G-B)$$

In this proposition, the unique undefeated equilibrium has one tranche with securities sold to all of the unconstrained investors. The truthful CRA places as many good assets as it can in the tranche, and if there is sufficient demand from unconstrained investors, includes bad assets as well. The price of the securities reflects the value and the perceived probability that the CRA is truthful. Because of the presence of the good assets, the issuer at both types of CRA generates strictly larger revenues than in the case of asymmetric information with no CRA. Once again, the opportunistic CRA does better than the truthful CRA.

For the last set of parameters, no CRA is hired:

Proposition 3 *If C1 and C3 do not hold, or if C1 holds and C2 does not hold, the unique undefeated equilibrium (which we denote $E_\varnothing$) has neither of the CRAs being hired.*

This follows directly from the proofs of Proposition 1 and Proposition 2. In this equilibrium the CRA can't generate value for the issuer, so the issuer does not hire the CRA and issues securities of value B, which are purchased by unconstrained investors.

3.2 Comparative Statics

We now look at comparative statics. We begin by looking at the parameters for which each equilibrium exists.

Lemma 5 *The equilibrium configuration is:*

1. *If $\bar{V}-g>B-b$: for $\theta_2\geq\frac{\bar{V}-B}{G-B}$, the equilibrium is E_{**}, for $\frac{\bar{V}-B}{G-B}>\theta_2\geq\frac{g-b}{G-B}$, the equilibrium is E_*, and for $\frac{g-b}{G-B}>\theta_2$, the equilibrium is $E_\varnothing$.*
2. *If $B-b\geq\bar{V}-g>0$: for $\theta_2\geq\frac{\bar{V}-B}{G-B}$, the equilibrium is E_{**}, for $\frac{\bar{V}-B}{G-B}>\theta_2$, the equilibrium is $E_\varnothing$.*

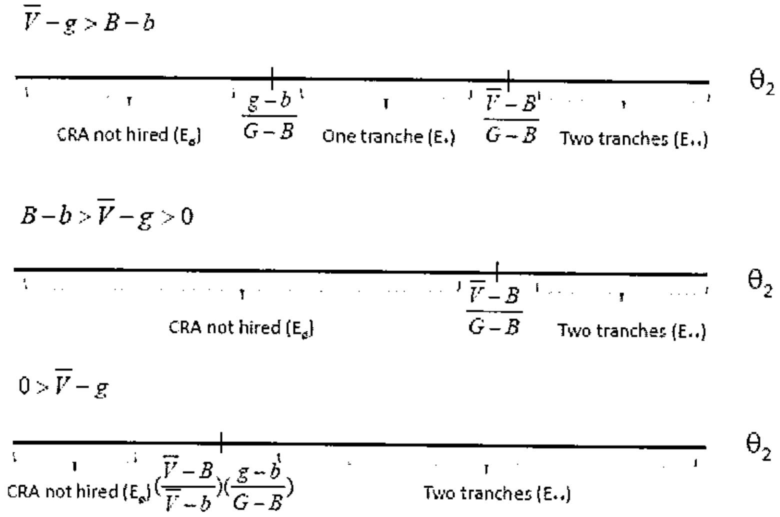

Figure 1: Equilibrium Configuration in Period Two

3. *If* $0 > \bar{V}-g$: *for* $\theta_2 \geq (\frac{\bar{V}-B}{\bar{V}-b})\frac{g-b}{G-B}$, *the equilibrium is* E_{**}, *for* $(\frac{\bar{V}-B}{\bar{V}-b})\frac{g-b}{G-B} > \theta_2$, *the equilibrium is* $E_{\varnothing}$.

We do not prove the lemma, as it follows directly from Propositions 1, 2, and 3. We illustrate the equilibrium configuration in Figure 1.

The lemma provides several insights. First, a one tranche equilibrium only exists if $\bar{V} - g > B - b$. This reflects the fact that the quality requirement of constrained investors is high relative to the benefit of retaining G assets and a tranche dedicated to constrained investors will not always be sustainable. It also means that the benefit to pushing B assets onto investors is not that large, which makes it desirable to sell off G assets to the unconstrained investors.

Second, the two tranche equilibrium exists when θ_2 is large. This means that it takes a substantial amount of reputation for honesty to be able to sell to constrained investors.

Third, the larger the quality requirement of constrained investors, the less likely it is that there will be a two tranche equilibrium.[11]

Fourth, securitization and the solicitation of credit ratings are strictly more likely to occur when investors' values for good assets are larger and are weakly more likely to occur when the quality requirement of constrained

[11] This can be found directly from the lemma by shifting $\bar{V}$.

investors is smaller or the issuer's value for retaining assets is smaller.[12] These make sense; securitization is more likely to occur when investors have a large demand and issuers have a low value for retention of good assets. The rating agency can help solve the asymmetric information problem and allow the issuer to transfer the good assets. There are not as many willing clients when constrained investors have high quality requirements, making securitization less likely.

Lemma 6 *If the equilibrium has two tranches, there is only rating inflation in the highly rated tranche. If the equilibrium has one tranche, there is rating inflation.*

For the equilibrium with two tranches, the top tranche's rating is equal to the value of what the truthful CRA is offering, which is substantially above the expected value given that the opportunistic CRA sells only bad assets. This is due to the scarcity of good assets and abundance of bad assets, as the truthful CRA can increase profits by selling the good assets to the constrained investors, even at a discount. The bottom tranche is rated bad, which reflects the quality of both the truthful and opportunistic CRA's assets perfectly.

We can quantify the degree of rating inflation by looking at the difference between the reported and actual value by the opportunistic CRA

$$R_C^O - V_C^O = (\bar{V} - B)/\theta_2$$

or, alternatively as the difference between the reported value and the price paid (P_C^O),

$$R_C^O - P_C^O = (1 - \theta_2)(\bar{V} - B)/\theta_2.$$

Therefore the degree of rating inflation increases with the difference between the constrained investors' minimum quality requirement and the value of bad assets, and the likelihood that the CRA is opportunistic. If we weight the degree of rating inflation by the measure of assets issued we obtain:

$$q_C^O \left(R_C^O - V_C^O\right) = \mu(G - B)$$

and

$$q_C^O \left(R_C^O - P_C^O\right) = (1 - \theta_2)\, \mu(G - B).$$

[12] By saying there is a "weak" preference, we mean that for each of the three parameter constellations in the lemma, there is a strict preference in at least one, and no negative preference.

For the one tranche equilibrium, the corresponding measures of rating inflation are given by a similar expressions:

$$R_U^O - V_U^O = \min\{1-\alpha, \mu\}(G-B)/(1-\alpha),$$

$$R_U^O - P_U^O = (1-\theta_2)\min\{1-\alpha, \mu\}(G-B)/(1-\alpha),$$

$$q_U^O(R_U^O - V_U^O) = \min\{1-\alpha, \mu\}(G-B),$$

$$q_U^O\left(R_U^O - P_U^O\right) = (1-\theta_2)\min\{1-\alpha, \mu\}(G-B).$$

It is clear that securitization improves welfare in the second period compared to the benchmark of no CRA, as otherwise the issuers would not hire the CRA.

From an ex-ante point of view (without knowledge of the type of the CRA), net total surplus in period 2 is measured by the quantity of good assets sold times $(G-g)$ plus the quantity of bad assets sold times $(B-b)$. For the two-tranche equilibrium E_{**}, net total surplus is equal to:

$$(G-g)\mu + (B-b)(1-\alpha+\mu(\theta_2 G + (1-\theta_2)B - \bar{V})/(\bar{V}-B).$$

This expression decreases in the fraction of constrained investors, α. This also also holds when there are no CRAs and asymmetric information, as the constrained investors are not willing to soak up residual assets. Net total surplus also increases in the probability of a CRA being truthful, θ_2, and in the measure of good assets, μ, as both increase the quantity of bad assets sold. Finally, net total surplus decreases with the minimum quality requirement of constrained investors, $\bar{V}$, as that reduces the possibility of selling assets.

The opportunistic CRA has no incentive to include good assets in the rated issue, as the quality of the assets are not observable or verifiable. CRAs state that their behavior is governed by reputation, i.e. the possibility of losing business in the future for taking advantage of investors today. In the next section, we examine how reputation will affect the opportunistic CRA and whether it can eliminate conflicts of interest in the first period.

4 The First Period

In this section, we will analyze equilibrium behavior in the first period, assuming conditions C1 and C2 hold such that the unique undefeated equilibrium is E_{**} in the last period. Of course, θ_2 is endogenous and determined by what happens in the first period, but we will later provide conditions on θ_1 such that θ_2 fulfills C1 and C2. More specifically, we will show that $\theta_1 \le \theta_2$

if the type of the CRA is not revealed in period one. Hence, if the above inequalities hold for the prior θ_1, then they also hold for the posterior, θ_2.

Denote the corresponding j-period profits for the CRA of type d, π_j^d. Hence, from Proposition 1:

$$\begin{aligned}\pi_2^O &= \left(\bar{V}-b\right)\mu\theta_2(G-B)/\left(\bar{V}-B\right) \\ &= \pi_2^T+\mu(g-b).\end{aligned}$$

Each CRA wants to maximize its expected discounted profits. For the opportunistic CRA, profits are $\Pi^O=\pi_1^O+\delta(1-p)\pi_2^O$. Here, π_1^O is first-period profits, p is a probability that the type of the CRA is discovered and δ is the discount factor. We define p such that $p=1$ if the CRAs separate in the first period, and otherwise $p=h(z)$. The argument z is a measure of how inflated (or inaccurate) the ratings are. We assume a functional form for this:

$$z=q_U(R_U-V_U^O)+q_C(R_C-V_C^O) \tag{5}$$

This represents the aggregate difference between reported and actual values for all securities issued. Note that the reports and quantities are not indexed by the type of CRA, since in a pooling equilibrium, both types must have the same report and quantity. The function h is strictly convex and continuously differentiable such that $h(0)=0, h'(0)=0$, and $h'(\mu(G-B))=\infty$. If there is no rating inflation at all, the opportunistic CRA is secure and will earn its full period two profits. If there is rating inflation and the opportunistic CRA is discovered, it is not hired in period 2. If the CRA's type is not revealed in period one, then the equilibrium posterior that it is truthful is $\theta_2=\theta_1/(\theta_1+(1-p)(1-\theta_1))$ in period two. From this formula follows immediately that in this case $\theta_1\leq\theta_2$. Given that an opportunistic CRA was not revealed in the first period, it is more likely that the CRA is truthful.

In the following proposition, we prove an important property of first period equilibria; namely that there are no separating equilibria.

Proposition 4 *The CRAs do not separate in the first period.*

We can thus restrict ourselves to looking only at pooling equilibria. In any pooling equilibrium, the opportunistic CRA's choice of assets to include in the tranches, (V_U^{O*},V_C^{O*}), must be optimal given the reports R_U^* and R_C^* and the quantities q_U^* and q_C^*, both of which are determined by the truthful CRA. More specifically, if the equilibrium has two tranches in the first period, then (V_U^{O*},V_C^{O*}) must be a solution to the following maximization problem (an analogous program applies if the equilibrium has only one tranche):

$$\max_{V_U^O \in [B,G], V_C^O \in [B,G]} \{q_U^*(\theta_1 R_U^* + (1-\theta_1)V_U^{O*} - (g-b)(V_U^O - B)/(G-B) - b) +$$
$$q_C^*(\theta_1 R_C^* + (1-\theta_1)V_C^{O*} - (g-b)(V_C^O - B)/(G-B) - b) +$$
$$(1 - h(q_U^*\left(R_U^* - V_U^O\right) + q_C^*\left(R_C^* - V_C^O\right)))\delta\mu(\theta_1/(\theta_1 + (1 - h(q_U^*\left(R_U^* - V_U^{O*}\right) +$$
$$q_C^*\left(R_C^* - V_C^{O*}\right)))(1-\theta_1)))\left(\bar{V} - b\right)(G-B)/\left(\bar{V} - B\right)\}$$

The first line represents the opportunistic CRA's profits from selling to unconstrained investors. Notice that the price paid depends on the beliefs of investors, which must be held fixed while finding the equilibrium choice. The cost to the CRA is opportunity cost of not holding the assets. The fraction of good assets is given by $\gamma_U^O = (V_U^O - B)/(G-B)$, which is used to derive this cost. The second line represents the opportunistic CRA's profits from selling to constrained investors. This is analogous to the first line. The third line is the probability the opportunistic CRA will have a business in the second period. This depends on the opportunistic CRA's choice, as more distortion from the reported value will lower the likelihood of survival. The fourth line is discounted payoff of an opportunistic CRA that is active in period two. The payoff here depends on the updated belief of investors given the period one action. This belief does not change with the CRA's unobservable choice.

In any pooling equilibrium, the first order conditions with respect to the securities issued in period 1 are given by

$$-\left(g-b\right)/(G-B) + \tag{6}$$
$$h'(z)\delta\mu(\theta_1/(\theta_1 + (1-h(z))(1-\theta_1)))\left(\bar{V} - b\right)(G-B)/\left(\bar{V} - B\right) \geq 0,$$

where the inequality can be replaced by an equality when $V_U^{O*} > B$ and $V_C^{O*} > B$.

If the equilibrium has only one tranche, $i \in \{U, C\}$, the same first-order condition applies, but it holds with equality whenever $V_i^{O*} > B$.

We will now focus on an equilibrium candidate, $\mathcal{E}_{**}$, where the two-tranche equilibrium E_{**} is played in the last period if the type of the CRA is not revealed in the first period, and the same type of equilibrium behavior is observed also in the first period. More specifically, in the first period the tranche for unconstrained investors will contain securities of value $R_U = V_U^O = V_U^T = B$ and the quantity of securities will equal the quantity of investors. The tranche for constrained investors will contain as many securities as possible with an expected value of $\bar{V}$. The truthful CRA will place all of its good assets and some bad assets in this tranche.

The assumption that $h'(\mu(G-B)) = \infty$ guarantees that (6) holds with equality in this equilibrium.[13] Hence, we can use the first-order condition to solve for z^*, the total amount of rating inflation chosen by the opportunistic CRA.

We will now provide sufficient conditions for the existence of $\mathcal{E}_{**}$.

$$\theta_1 G + (1-\theta_1)B \geq \bar{V} \tag{C1'}$$

$$\left(\bar{V}-b\right)\mu\theta_1(G-B)/\left(\bar{V}-B\right) - \mu(g-b) > 0 \tag{C2'}$$

Condition C1' is analogous to condition C1 and implies that in the first period, it is feasible to sell to constrained investors. Condition C2' is similar to condition C2 in that it implies that first period profits for the truthful CRA will be positive. Note that as $\theta_1 \leq \theta_2$ if the type of the CRA is not revealed in the first period, C1 and C2 are implied by these conditions. Also note that these conditions are sufficient, but not necessary; in the first period the opportunistic CRA will issue a security worth more than B to constrained investors and actual profits for the truthful CRA will be higher than the expression in C2'.[14]

In the following proposition, we show that $\mathcal{E}_{**}$ is an equilibrium.

Proposition 5 *If C1' and C2' hold, then $\mathcal{E}_{**}$ can be sustained as an equilibrium.*

The following lemma describes how rating inflation changes with the parameters in $\mathcal{E}_{**}$.

Lemma 7 *The total amount of rating inflation by the opportunistic CRA in period 1 is:*

1. *Increasing in g and $\bar{V}$*
2. *Decreasing in δ, G, and θ_1*
3. *Increasing in B if $2\bar{V} > G$ and decreasing in B if $2\bar{V} > G$*
4. *Increasing in b if $g > \bar{V}$ and decreasing in b if $g < \bar{V}$.*

[13]This can be demonstrated using the solution $V_C^{O*} = \frac{(\mu(G-B)-z^*)\bar{V}+\theta_1 B z^*}{\mu(G-B)-(1-\theta_1)z^*}$, which we derive below.

[14]Technically, profits in the first period for the truthful CRA could be negative as long as it has positive second period profits. The benefit of condition C2' is that it implies profits are also positive in the second period (given that $\theta_2 \geq \theta_1$).

Rating inflation increases if there is a larger payoff to retaining good assets. It also increases if it is more costly to satisfy constrained investors. This occurs because second period profits are decreasing in the quality requirement of constrained investors, as it is more difficult to push securities onto them. As second period profits decline, the benefit of not inflating ratings dissipates.

Rating inflation decreases if reputation is more important (proxied for by the discount factor), if it can get more for its good assets, and if the prior that the CRA is truthful in period 1 is larger. The insight on the prior comes from the fact that the more likely the period 1 CRA is truthful, the more there is to gain for the opportunistic CRA in period 2, implying it will choose less rating inflation in period 1 to increase the chance of survival.

We will now fully characterize the equilibrium. The tranche for the constrained investors is characterized by two expressions:

$$\theta_1 R_C + (1-\theta_1)(R_C - z^*/q_C) = \bar{V}$$
$$q_C R_C = \mu G + (q_C - \mu)B$$

The first expression says that the expected value of the constrained tranche will be equal to $\bar{V}$. With probability θ_1, the CRA is truthful and the report is equal to the value. With probability $1-\theta_1$, the CRA is opportunistic and can choose to inflate ratings. The expected value depends the CRAs choice z^*, and the expression for that value is derived from equation (5) and the fact that $R_U = V_U^O = B$. The second expression defines q_C from the choice of the truthful CRA: the total value of the truthful CRA's pool of assets for the constrained tranche is equal to the total amount of G assets it has and the number of B assets it includes.

There are two unknowns in the two equations above, q_C and R_C. Solving, we obtain the following:

$$\begin{aligned} R_C^* &= \frac{\mu(G-B)\bar{V} - (1-\theta_1)Bz^*}{\mu(G-B) - (1-\theta_1)z^*}, \\ q_C^* &= \frac{\mu(G-B) - (1-\theta_1)z^*}{\bar{V} - B}. \end{aligned}$$

Using this, we can obtain expressions for the value of the securities issued by the opportunistic CRA and the rating inflation:

$$\begin{aligned} V_C^{O*} &= \frac{(\mu(G-B) - z^*)\bar{V} + \theta_1 B z^*}{\mu(G-B) - (1-\theta_1)z^*}, \\ R_C^* - V_C^{O*} &= z^* \frac{\bar{V} - B}{\mu(G-B) - (1-\theta_1)z^*}. \end{aligned}$$

First period profits for the opportunistic CRA are given by:

$$q_C^*(\bar{V} - b) + \frac{g - b}{G - B} z^* - \mu(g - b)$$

First period profits for the truthful CRA are given by:

$$q_C^*(\bar{V} - b) - \mu(g - b)$$

5 Conclusion

In this paper we examine the interaction between security design, credit rating agencies, and investor clienteles. This is particularly important in the wake of the poor performance of ratings for structured products.

We model rating agencies as long lived players with incentives driven by reputation. They structure products with issuers for constrained and unconstrained investors. The presence of constrained investors provides a new motivation for the pooling and tranching of assets; catering to a specific clientele. We find that when quality requirements for constrained investors are higher, the CRA's profits decrease as it is harder to place assets. Rating inflation increases in response, as lower future profits imply more incentives to take advantage of current investors.

We also demonstrate that reputational incentives affect security design in the form of the number of tranches. Rating inflation occurs in the top tranches and will depend on negatively on the belief that the CRA is truthful because of the tradeoff between milking reputation now or later.

There are several future avenues of research to explore. We would like to look further into first period dynamics. It would also be of interest to add risk (and risk aversion) to the model, to relate our results to others in the literature. Furthermore, we would also like to examine the role of competition and shopping in this environment.

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6 Appendix

6.1 Proof of Lemma 1

Consider first selling only to unconstrained investors. The net payoff in (3) is increasing in q_U, so the issuer will set $q_U = 1 - \alpha$ and sell to all unconstrained investors. Furthermore, the expression is increasing in γ_U if $G - g > B - b$ and decreasing in γ_U if $G - g < B - b$. Therefore if $G - g > B - b$, the issuer will set $\gamma_U = \min\{1, \frac{\mu}{1-\alpha}\}$. This yields a net payoff of:

$$\min\{1-\alpha, \mu\}(G - g - B + b)) + (1-\alpha)(B - b). \tag{7}$$

If $G - g < B - b$, the issuer will set $\gamma_U = 0$, as the issuer can sell all B assets to the unconstrained investors, given that $M \geq 2$. This yields a net payoff of

$$(1-\alpha)(B - b). \tag{8}$$

The same net payoff entails if $G - g = B - b$.

Now consider the case of selling to both types of investor. If securities worth $V > \bar{V}$ were sold to constrained investors, the issuer could always add more bad assets to the constrained portfolio and increase its profits. Therefore, securities sold to constrained investors will be worth $\bar{V}$ to them,

$$\gamma'_C G + (1 - \gamma'_C)B = \bar{V}. \tag{9}$$

For the same reason, the issuer will set $q_U = 1 - \alpha$. Finally, it is easy to see that the net payoff is maximized if all good assets are allocated to the constrained tranche, so that $q'_C \gamma'_C = \mu$ given A1. This implies that $q'_C = \mu(G - B)/\left(\bar{V} - B\right)$ and that $\gamma'_U = 0$, $q'_U = 1 - \alpha$, and $\gamma'_I = 0$. The maximum net payoff selling to both types is thus:

$$(1-\alpha)(B - b) + (G - g)\mu + (B - b)\mu(G - \bar{V})/\left(\bar{V} - B\right). \tag{10}$$

As the issuer is able to sell all of the good assets at their maximum value and more than $1 - \alpha$ of the bad assets at their maximum value, the payoff from selling to both types of investor is strictly larger than the payoff of selling to just unconstrained investors.

6.2 Proof of Lemma 2

First, consider the issuer announcing the full information equilibrium allocation, i.e. $\mu(G - B)/\left(\bar{V} - B\right)$ securities worth $\bar{V}$ and $1 - \alpha$ securities worth B, where it retains the rest. This can't be an equilibrium, as the issuer has the incentive to substitute its retained assets for the assets in the portfolio meant for constrained investors. It will be unable to sell to constrained investors at all in fact, because if at some point constrained investors believe they are getting a high quality security, the issuer will move more bad assets into the portfolio. The issuer is also unable to sell securities with a value greater than B to unconstrained investors, as the issuer can't commit not to substitute something more B assets into the portfolio. Therefore, the only possible allocation is selling securities of value B to the $1-\alpha$ unconstrained investors. The payoff for the issuer is $\mu g + (1-\alpha)B + (M - \mu - (1-\alpha))b$.

6.3 Proof of Lemma 3

Suppose there is an equilibrium where the CRA is hired where this is not true and the price of a security is $p \geq B$. Then, since $B < g$, the opportunistic CRA could gain by retaining the good assets backing the tranches and replacing them with bad assets (recall that there are always enough bad assets to do this since $M \geq 2$) without changing its reported values.

6.4 Proof of Lemma 4

In a separating equilibrium, the type of each CRA would be revealed perfectly. Hence, the opportunistic CRA would only be able to issue securities worth B, by Lemma 3, and it would thus only be able to sell to unconstrained investors. Since it could not create any value, it would not be hired by the issuer.

The truthful CRA could not be issuing securities resulting in a positive surplus, or the opportunistic CRA would have a profitable deviation by mimicking the sizes and ratings of its issues. Furthermore, if it were issuing securities worth B, it would not be hired.

6.5 Undefeated Equilibria: Definition and Application

In this subsection, we define the concept of Undefeated Equilibria, as put forth by Mailath, Okuno-Fujiwara, and Postlewaite (1993). We begin with the definition of a Pure Strategy Sequential Equilibrium (PSE).

In addition to the notation in section 3.1, we add the following. Denote an arbitrary CRA type by d and the set of such types by $D = \{T, O\}$. Let $p = (p_U, p_C)$. The profits to the CRA of type d are denoted by $u(s, p, d)$. Finally, define the probability function $\Theta(d)$ such that $\Theta(T) = \theta$ and $\Theta(O) = 1 - \theta$.

Definition 6 *A strategy profile* $E^* = (s^*, p^*, \beta^*)$ *is a **Pure Strategy Sequential Equilibrium (PSE)** if and only if:*

1. $\forall d \in D : s^*(d) \in \arg\max_{s \in S_d} u(s, p, d)$,
2. $\forall m \in M : p_U(m) = v_U^\beta(m)$, *and* $p_C(m) = v_C^\beta(m)$ *if* $v_U^\beta(m) \geq \bar{V}$ *and* $p_C(m) = 0$ *otherwise,*

3. $\forall d \in D$ and $\forall m \in M : \beta^*(d|m) = \Theta(d)1_{m(d)=m} / \sum_{d' \in D} \Theta(d')1_{m(d)=m}$ if the denominator is positive, where $1_{m(d)=m}$ is an indicator function that takes the value 1 if $m(d) = m$ and 0 otherwise.

In words, a strategy profile is a Pure Strategy Sequential Equilibrium if: 1. each type of CRA is using pure strategy maximizing profits given the investors' bids and the other CRA's strategy, 2. each investor bids his expected value conditional upon observed tranche sizes and reported values, 3. beliefs are calculated using Bayes' rule for tranche sizes and reported values used with positive probability.

Definition 7 *A PSE, $E = (s, p, \beta)$, **defeats** another PSE, $E' = (s', p', \beta')$, if and only if:*

1. $\forall d \in D : m'(d) \neq m$ and $K = \{d \in D : m(d) = m\} \neq \emptyset$,
2. $\forall d \in K : u(s, p, d) \geq u(s', p', d)$ and $\exists d \in K : u(s, p, d) > u(s', p', d)$,
3. $\exists d \in K : \beta'(d|m) \neq \Theta(d)\pi(d) / \sum_{d' \in D} \Theta(d')\pi(d')$ for some $\pi : D \to [0, 1]$ satisfying:
   $d' \in K$ and $u(s', p', d') < u(s, p, d') \Rightarrow \pi(d') = 1$, and
   $d' \notin K \Rightarrow \pi(d') = 0$.

In words, an equilibrium E defeats another equilibrium E' if: 1. there is a message m sent only in E, 2. the set of types K who send this message are all better off in E than in E' and at least one of them is strictly better off, and 3. under E', the beliefs about some such a type are not a posterior off-the-equilibrium path assuming only types in K send m and that they do so with probability one if they are strictly worse off than under E.

A PSE is said to be *undefeated* if the game has no other PSE that defeats it.

In order to apply the undefeated concept, we define a *strictly Pareto dominant equilibrium* as a PSE that has strictly higher payoff for both types of CRAs than any PSE with a different strategy profile.

Lemma 8 *If a strictly Pareto dominant equilibrium exists, then it defeats any PSE with a different strategy profile.*

Proof. If the game has a unique PSE the proof is trivial. Suppose therefore that the game has a strictly Pareto dominant equilibrium, E, and a PSE with a different strategy profile, E'. First note that by Lemma 4, both must be pooling (although the CRAs may not be hired in one of them). Second, since the truthful CRA is restricted to honest reports, the messages sent in in the two equilibria must be different $m \neq m'$(if the CRAs are not hired in one of the equilibria, the corresponding message is empty). Third, both CRAs are strictly better off under E than under E'. Finally, beliefs in E' given the message m cannot be a posterior assuming the truthful CRA sends this message with probility one, or there would be profitable unilateral deviation for both types of CRAs. Hence, E defeats E'. ■

Therefore, it suffices to find a strictly Pareto dominant equilibrium.

6.6 Proof of Proposition 1

We begin by finding the equilibrium with two tranches that maximizes the profits of the truthful CRA.

It is clear that a necessary condition for the equilibrium is that $\theta_2 G + (1-\theta_2)B \geq \bar{V}$, since otherwise the constrained investors cannot be served.

To make the notation slightly easier, let μ_U and μ_C be the measure of good securities backing the unconstrained and constrained tranches of the truthful CRA, and let ν_U and ν_C be the measure of bad securities backing the unconstrained and constrained tranches of the truthful CRA. Therefore mapping this to our notation in the text, $\mu_j = q_j\gamma_j^T$ and $\nu_j = q_j(1-\gamma_j^T)$, where $j \in \{U, C\}$.

Using this, and Lemma 3, we can write the net profits in any possible pooling equilibrium as:

$$\theta_2(\mu_U G + \nu_U B) + (1-\theta_2)B(\mu_U + \nu_U) - \mu_U g - \nu_U b +$$

$$\theta_2(\mu_C G + \nu_C B) + (1-\theta_2)B(\mu_C + \nu_C) - \mu_C g - \nu_C b - (1-\alpha)(B-b)$$

The first line is the payoff from selling to unconstrained investors rather than retaining the assets. The second line is the payoff from selling to constrained investors rather than retaining the assets, minus the payoff an issuer could achieve without the CRA. We simplify this expression:

$$\mu_U(\theta_2 G+(1-\theta_2)B-g)+\nu_U(B-b)+\mu_C(\theta_2 G+(1-\theta_2)B-g)+\nu_C(B-b)-(1-\alpha)(B-b).$$

This should be maximized with respect to $\mu_U, \mu_C, \nu_U, \nu_C$ given the restrictions:

$$0 \leq \mu_U + \nu_U \leq 1-\alpha,$$

$$0 \leq \mu_C + \nu_C \leq \alpha,$$

$$0 \leq \mu_U + \mu_C \leq \mu,$$

$$\mu_C(\theta_2 G + (1-\theta_2)B - \bar{V})/(\bar{V} - B) \geq \nu_C.$$

The first two inequalities say that the amount of securities sold to each type of investor can't exceed the amount of each type of investor. The third inequality states that the amount of good assets sold can't surpass the amount of good assets available. The last inequality represents the fact that constrained investors demand securities with expected value of at least $\bar{V}$.

It is clear that any solution has $\mu_U + \nu_U = 1 - \alpha$, since otherwise profits can always be increased by increasing ν_U without affecting any of the other constraints.

Any solution must have the second constraint not binding, as assumption A1 tells us it is impossible given the scarcity of good assets to sell to all constrained investors. Without the second constraint, the fourth constraint will bind since the amount of bad assets for the constrained, ν_C ,will be increased until the minimum quality requirement of the constrained investors is binding. This yields:

$$\mu_C(G\theta_2 + (1-\theta_2)B - \bar{V})/(\bar{V} - B) = \nu_C.$$

Substituting the binding constraints into the expression for net profits gives:

$$\mu_U(\theta_2 G + (1-\theta_2)B - g) + (1 - \alpha - \mu_U)(B - b) + \mu_C(\theta_2 G + (1-\theta_2)B - g) +$$

$$\mu_C(B - b)(\theta_2 G + (1-\theta_2)B - \bar{V})/(\bar{V} - B) - (1-\alpha)(B - b)$$

It is clear that μ_U has a strictly lower partial derivative than μ_C.

Hence, if the partial derivative with respect to μ_C is positive, the solution has $\mu_U = 0$, $\mu_C = \mu$, $\nu_U = 1 - \alpha$, and $\nu_C = \mu(\theta_2 G + (1-\theta_2)B - \bar{V})/(\bar{V} - B)$. This clearly implies a positive profit, and therefore the CRA would be hired.

If the partial with respect to μ_C is negative, then the partial with respect μ_U is also negative, and hence $\mu_U = 0$, $\mu_C = 0$, $\nu_U = 1 - \alpha$, and $\nu_C = 0$. Since this implies zero profits, excluding rating costs, the CRA would not be hired.

If the partial with respect to μ_C is zero, then the partial with respect μ_U is negative, and hence (c) $\mu_U = 0$, $\mu_C \leq \mu$, $\nu_U = 1 - \alpha$, and $\nu_C = \mu_C(\theta_2 G + (1-\theta_2)B - \bar{V})/(\bar{V} - B)$. Since the profits are zero excluding rating costs, the CRA would not be hired.

It is easy to see that the solutions can be implemented as equilibria. Just assume beliefs are equal to the prior for any out-of-equilibrium message. The truthful CRA then has no incentive to deviate, since the equilibrium is maximizing net profits. Moreover, the opportunistic CRA is also weakly better off than for any other message.

Finally, the above equilibrium maximizes the profits of the truthful CRA. If we denote the truthful CRA's profits are π_T , then the profits for the opportunistic CRA are $\pi_T + (\mu_U + \mu_C)(g - b)$. Since the partial of profits with respect to μ_C is even higher than for the truthful CRA, if the equilibrium is payoff maximizing for the truthful CRA, then so is it for the opportunistic CRA. Therefore it is a Pareto Dominant equilibrium, which by Lemma 8 means that it is undefeated.

Lastly, we can solve for the report and quantity issued for the constrained tranche (going back to our notation in the text) using the following two equations:

$$\theta_2 R_C + (1-\theta_2)B = \bar{V}$$
$$q_C R_C = \mu G + (q_C - \mu)B,$$

The first expression says that the expected value of the constrained tranche will be equal to $\bar{V}$. The second expression defines q_C from the choice of the truthful CRA: the total value of the truthful CRA's pool of assets for the constrained tranche is equal to the total amount of G assets it has and the number of B assets it includes.

There are two unknowns in the two equations above, q_C and R_C. Solving, we obtain the following:

$$\begin{aligned} R_C &= \left(\bar{V} - (1-\theta_2)B\right)/\theta_2 \\ q_C &= \left(\theta_2 \mu (G-B)\right)/\left(\bar{V} - B\right). \end{aligned}$$

6.7 Proof of Proposition 2

As with the previous proposition, we begin by finding the equilibrium that maximizes the profits of the truthful CRA.

If $\theta_2 G + (1-\theta_2)B < \bar{V}$, the truthful CRA cannot sell to the constrained investors. If the the restriction $\mu_C = 0$ is added to the optimization problem in the proof of Proposition 1, the solution has $\mu_U = \min\{1-\alpha, \mu\}$, $\mu_C = 0$, $\nu_U = 1 - \alpha - \mu_U$, and $\nu_C = 0$ if the partial with respect to μ_U, $\theta_2(G-B) - (g-b)$, is positive. For sufficiently small rating cost, this is clearly payoff superior to issuing $1-\alpha$ securities worth B to unconstrained investors, and hence the CRA will be hired.

If the partial is negative, then the solution has $\mu_U = 0$, $\mu_C = 0$, $\nu_U = 1-\alpha$, and $\nu_C = 0$. However, in this case, due to the cost of rating, the CRA would not be hired.

If the partial is zero, then the solution has $\mu_U \leq \min\{1-\alpha, \mu\}$, $\mu_C = 0$, $\nu_U = 1 - \alpha - \mu_U$, and $\nu_C = 0$. However, since the payoff is the same as in the solution with negative partial, the CRA would not be hired.

It is easy to see that the first solution can be implemented as an equilibrium. Just assume beliefs are equal to the prior for any out-of-equilibrium message. The truthful CRA then has no incentive to deviate, since the equilibrium is maximizing net profits. Moreover, the opportunistic CRA is also weakly better off than for any other message.

Finally, the above equilibrium maximizes the profits of the truthful CRA. If we denote the truthful CRA's profits are π_T , then the profits for the opportunistic CRA are $\pi_T + \mu_U(g-b)$. Since the partial of profits with respect to μ_C is even higher than for the truthful CRA, if the equilibrium is payoff maximizing for the truthful CRA, then so is it for the opportunistic CRA. Therefore it is a Pareto Dominant equilibrium, which by Lemma 8 means that it is undefeated.

6.8 Proof of Proposition 4

In a separating equilibrium, the type of each CRA would be revealed perfectly. Hence, by Lemma 3 the opportunistic CRA would only be able to issue securities worth B in period two, and it would thus not be hired then. This implies that it has no reputation concerns and would never issue a security worth more than B in period one either, and as it is separating in the first period, it would not be hired in the first period either.

The truthful CRA could not issue securities in period one resulting in a positive surplus on its own, or the opportunistic CRA would have a profitable deviation by mimicking the sizes and ratings of its issues (with actual values equal to or lower than the reported). Furthermore, if it were issuing securities worth B, it would not be hired.

6.9 Proof of Lemma 7

Define the function $k_{\theta_1}()$ implicitly by

$$h'(z)(\theta_1/(\theta_1 + (1-h(z))(1-\theta_1))) = k_{\theta_1}^{-1}(z). \tag{11}$$

The left hand side of this equation is increasing in both θ_1 and z. This implies the function $k_{\theta_1}()$ is increasing in z and decreasing in θ_1. Using this function and the first-order condition, we can solve for z:

$$z^* = k_{\theta_1}\left(\frac{(g-b)(\bar{V}-B)}{\delta(G-B)^2(\bar{V}-b)}\right) \tag{12}$$

It follows that z^* is increasing in g and $\bar{V}$ and decreasing in δ, G, and θ_1. It is increasing in B if $2\bar{V} > G$ and decreasing in B if $2\bar{V} > G$. It is increasing in b if $g > \bar{V}$ and decreasing in b if $g < \bar{V}$.

6.10 Proof of Proposition 5

Assume out-of-equilibrium path beliefs assigning probability one to the CRA being opportunistic. We know from the above that $\theta_1 \leq \theta_2$ if the type of the CRA is not revealed in period 1. Hence, C1' implies C1 and C2' implies C2. This guarantees that E_{**} is a unique undefeated equilibrium of the game in the second period and that both types of CRAs earn positive profits then if no type is revealed in the first period.

In the first period, the opportunistic is issuing securities worth $V_U^O \geq B$ to constrained investors. Hence, by C1' such investors can be served. Moreover, the fact that $V_U^O \geq B$ also implies that the first-period profit of the truthful CRA will be larger than $\left(\bar{V} - b\right) \mu\theta_1(G - B)/\left(\bar{V} - B\right) - \mu(g - b)$, and hence positive by C2'.

Consider a deviation by the truthful CRA in the first period. Such a deviation would, according to the beliefs assumed, imply that the CRA is not hired in any of the periods and hence earns a profit of zero.

The opportunistic CRA would not deviate by changing its value V_U^O or V_C^O as these are optimally chosen in the maximization problem. It will also not deviate by chosing a different message given the off-the-equilibrium-path beliefs.