Nasdaq

Nasdaq Cash Flow Margin

Margin methodology guide for Nordic fixed income products

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Document History

Revision	Published	Author(s)	Summary of Changes
1.1	2022-07-07	Christian Corfitsen	Updated template.
1.2	2022-09-07	Simon Östlund	Review, inclusion of SWESTR OIS and NOWA futures.
1.3	2023-01-26	Andreas Strandberg Kieri	Inclusion of EUR IRS and OIS.

Executive Summary

This document describes the Nasdaq Clearing Cash Flow Margin (CFM) methodology for calculating initial margin for fixed income instruments.

CFM is a curve-based margin methodology, which uses three principal components, PCs, to generate scenarios of stress.

CFM encompasses three different curve building techniques. These are a discount factor curve interpolated with cubic splines, a linearly interpolated forward fixing rate curve, and a piecewise constant instantaneous forward rate curve.

CFM uses three kinds of valuation techniques. For calculating the market values of the semi-standardized derivatives (e.g. bond forwards & IMM-FRAs) a simple formula based approach is used. A dual-curve methodology is used for cash-collateralized OTC rates derivatives (e.g. Swaps, FRAs & OIS) where the market value is shifted on a daily basis. For the valuation of repos and for margining purposes, the discount factor curves are used in a one-curve valuation framework.

CFM enables the configuration of correlation of curve stress applied to different yield curves.

CFM covers all of the fixed income products cleared by Nasdaq Clearing, including government and mortgage bond forwards, IMM-FRA, RIBA futures, STIBOR futures, CIBOR futures, Repo transactions, Interest Rate Swaps, tailor made FRA, and Overnight Index Swaps.

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1. Purpose of CFM – Calculate margin requirement

The goal of CFM, and of any margin methodology, is to calculate margin requirements for cleared derivative positions, and in CFM's case specifically for fixed income instruments. A customer of the clearinghouse that has cleared such instruments needs to pledge collateral, the size of which relates to the risk in its positions. Among the wanted qualities of a margin methodology is the ability to calculate margin requirement on trade, position and portfolio level which stands in relation to the risk level.

1.1. Different kinds of margin constituents

When discussing the margin requirement of a specific instrument, it is important to keep in mind that it might be composed of a number of constituents. The most important of these are the market value and the initial margin of the position. At certain points in time during the life-time of a cleared fixed income position it can also generate payment margin and delivery margin.

The instrument subject to the margin requirement decides which margin components to include in each respective margin requirement calculation.

1.2. Different kinds of fixed income instruments

Nasdaq offers clearing of a wide range of fixed income instruments. Within this offering a range of different market models co-exists. They can be described as standardized contracts, generic rates instrument (OTC) and repo transactions.

1.2.1. Standardized contracts

Representing the Clearinghouse's original offering in fixed income, these instruments share the trait of having a formula based market valuation and highly standardized definitions with termination dates occurring quarterly. The group includes IMM-FRAs, futures, hybrid forwards with monthly cash settlement, and options on FRAs and forwards.

For these contracts the margin requirement can include any of the margin components listed above; the aggregated P/L (i.e. the market value), the initial margin and in connection to settlement dates and termination dates also payment and delivery margin

1.2.2. Generic rates instruments

Nasdaq offers clearing of OTC fixed income instruments such as interest rate swaps, tailor made FRAs and OIS. The clearinghouse's margin model for these instruments is that they are mutually cash-collateralized on a daily basis and that an initial margin is demanded which can be pledged in any approved security.

Therefore, for generic rates instruments the margin requirement is equal to the initial margin. No payment or delivery margin is calculated for these types of instruments.

1.2.3. Repo transactions

The Clearinghouse's repo clearing service demands that its participants pledge both the market value and the initial margin of each repo transaction. Thus the margin requirement includes the market value and the initial margin. For repo transactions no payment or delivery margin is demanded.

1.2.4. Conclusion

CFM is a margin methodology that operates on all fixed income instruments, and it therefore needs to take account of the various market models that exist within Nasdaq's offering. For standardized contracts it calculates all margin constituents. For Repo transactions and Generic Rates Instruments it calculates the market value and the initial margin, although the market value isn't included in the margin requirement for the Generic Rates Instruments.

With the conclusion of this overview of Nasdaq's fixed income clearing offering in mind, the following chapters will turn to the key concepts of CFM.

2. Key concept 1 – Cash flows

As is implied by its name, at the heart of CFM lays the concept of cash flows. All fixed income instruments are represented as a collection of cash flows when the margin requirement is calculated. Each instrument's NPV in a given market scenario is defined as the sum of the NPVs of all of its cash flows.

2.1. Types of cash flows

CFM deals with two types of cash flows. These are fixed cash flows and floating cash flows.

2.1.1. Fixed cash flows

A cash flow whose size (as measured in monetary terms in the currency of the instrument) is constant and known before hand is considered to be a fixed cash flow. Fixed cash flows can be any of the following:

- Bond notional payments on maturity
- Bond or swap coupons
- Contracted rates in forwards and futures on deposit rates
- Considerations in repo transactions, i.e. the cash payments of repo legs.
- Previously floating cash flows that have been fixed.

In general, fixed cash flows that occur in the future will be discounted down to today when calculating their NPVs.

2.1.2. Floating cash flows

A cash flow whose size is still unknown is considered as a floating cash flow. The size of the cash flow will be determined at some pre-defined point in time in the future. Floating cash flows are mainly associated with derivatives of deposit rates, i.e. swaps, FRAs and RIBAs.

The NPVs of floating cash flows will be calculated through first forecasting its size, and then discounting that value down to today.

2.1.3. Conclusion

An integral part of CFM, the cash flows are generated from instrument data and used in valuation of instruments in various market scenarios. To gain a full picture of an instrument's market value also the yield curves, which act as price bearers, are needed for forecasting floating cash flows and discounting all cash flows.

3. Key concept 2 – Curves

The second integral part of CFM is the concept of curves acting as price bearers. In fixed income markets there generally are sets of instruments with the same underlying credit risk, but with differing maturities in time. For a given creditor, it is therefore natural to represent the relation between price (or interest rate) on one hand, and time on the other, as a curve in a two-dimensional graph. The curve acts as a price bearer which carries market information that can be used for the valuation of fixed income derivatives.

3.1. Different kinds of curves

CFM uses three kinds of curves for the calculation of NPV for the different instruments it clears.

3.1.1. Discount factor curves

A discount factor curve expresses the present value of cash flows occurring at various times in the future.

More formally, the discount factor function, $d(0,m_T)$, is defined as the price today of a zero-coupon bond that pays 1 at the value date, T. The discount factor is a decreasing function of time to maturity, $m_T=(T-t)/365$, and by definition it starts at 1 as the present value of a cash flow occurring today, t, is equal to its notional size.

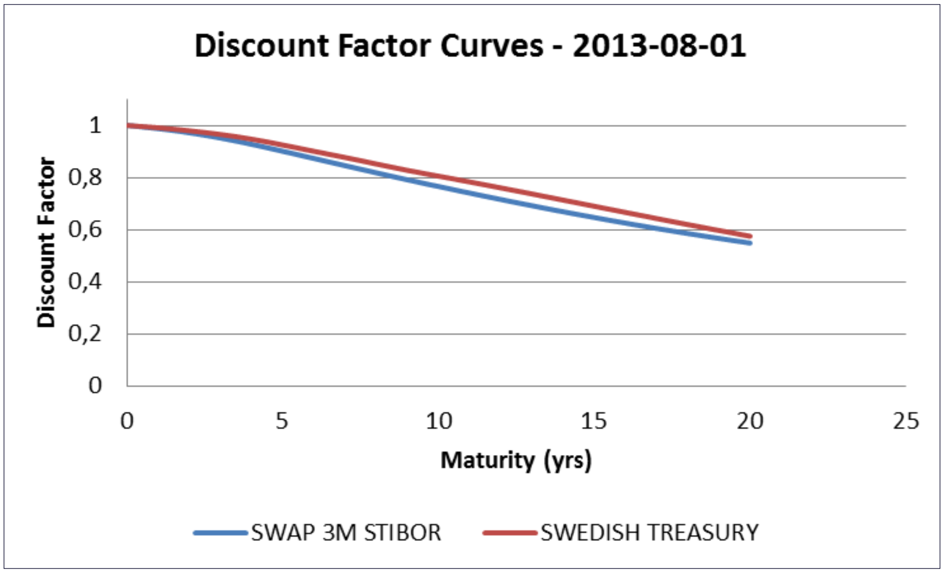


Figure 1. Two examples of discount factor curves.

Discount factor curves are used in the market valuation of repo transactions.

Discount factor curves are used for the calculation of margins for all instruments in CFM. As will be explained in greater detail below, the margin scenarios are generated through stressing yield curves. Therefore the discount factor curves are transformed into yield curves when used in this context.

The yield curves are expressed as a spot rate function, $i(0,m_T)$, that is defined as the yearly compounded interest rate for a zero-coupon bond that is traded today, and that matures at the value date, T. The equation below relates the spot rate function to the discount factor function.

$$i(0,m_T) = \left(\frac{1}{d(0,m_T)}\right)^{\frac{1}{m_T}} - 1$$

Nasdaq expresses all yield curves as yearly compounded interest rates with day count convention.

$\frac{ACT}{365}$.

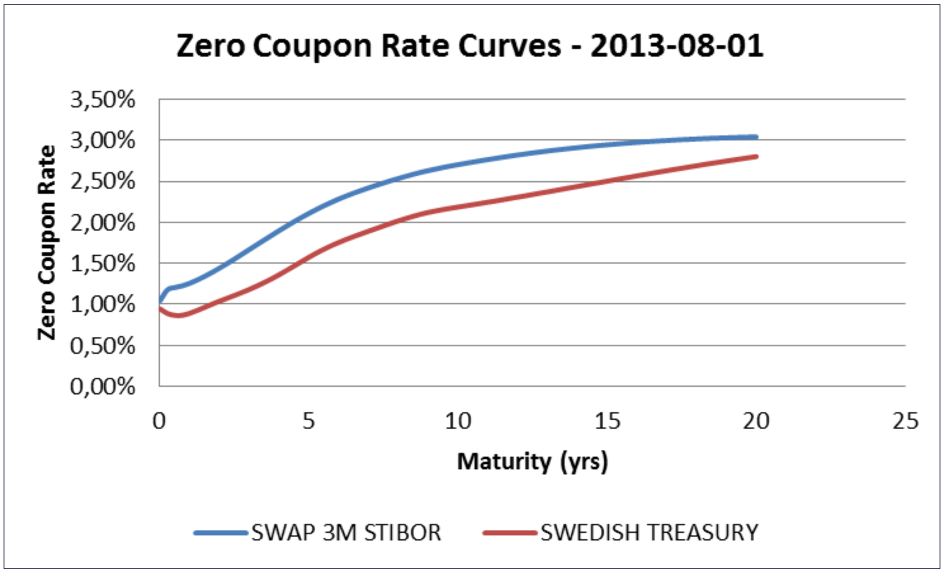


Figure 2. Two examples of discount factor curves.

In margin calculations, Nasdaq uses these kind of curves not only for discounting, but also for forecasting floating rates. This is done through calculating a forward rate function. The forward rate function, $f(0,m_t,m_T)$, is defined as the yearly compounded implied forward rate on an investment that starts at the settlement date, t and ends at the value date, T.

The equation below relates the forward rate function to the spot rate function.

$$f(0,m_t,m_T) = \left(\frac{(1+i(0,m_T))^{m_T}}{(1+i(0,m_t))^{m_t}}\right)^{\frac{1}{m_T-m_t}} - 1$$

3.1.2. Forward fixing rate curves

A second type of curve is the Forward Fixing Rate curve which is used when forecasting floating cash flows in the valuation of Generic Rates Instruments.

The Forward Fixing Rate function, F(T), is defined as the expected fixing rate for a future deposit starting on the start date T.

Every forecasting curve describes the future fixings of a deposit of a specific tenor. As an example, when forecasting a 3M STIBOR fixing, a separate curve will be used as compared to when forecasting a 6M STIBOR fixing.

The Forward Fixing Rate curve is used in the valuation of Generic Rates Instruments.

The Forward Fixing Rate curve is not used in margin calculations. Instead a discount factor curve constructed using FRAs and Swaps is used for forecasting future deposit rates when calculating margins.

3.1.3. Instantaneous forward rate curves

The last type of curve that is implemented in CFM is the instantaneous forward rate curve.

The instantaneous forward rate function, f(t), defines the interest rate for an infinitesimal time period starting at time t. Through integrating this function over time, the interest rate amount for different time periods can be calculated. Specifically, the related discount factor for a specific point in time can be calculated as:

$$d(0,T) = e^{-\int_0^T f(t)\,dt}$$

CFM uses this kind of curve for the valuation of Overnight Index Swaps in different currencies. In this particular case the same curve is used both for discounting and forecasting.

The instantaneous forward rate curve is not used in margin calculations. Instead a discount factor curve constructed using OIS as price carriers is used when calculating the margin requirement for a cleared OIS.

3.1.4. Conclusion

The three curves used by CFM have been introduced. More information about how they are constructed from the market prices of calibration instruments can be found in the appendix on bootstrapping. The market information represented in the curves is applied to the cash flow collections defining each instrument when calculating the margin requirements.

4. Key Concept 3 - Principal components analysis

The present value of a future cash flow exposed to a given yield curve will change if the shape of the yield curve changes. A yield curve may change in numerous ways, but there is empirical evidence that the curve's first three principal components express the vast majority of the changes.

This section defines the first three principal components from an economical point of view. It also gives an overview of how CFM uses the principal components to produce the stressed yield curve scenarios which are used in the margin calculations.

4.1. Definition of the principal components

Principal components (PC) are defined as independent (uncorrelated) changes of the yield curve.

4.1.1. PC1: Parallel shift

For a yield curve the first PC is a parallel shift of the entire curve. This PC usually explains 75%-85% of the curve's historical movement. This is also quite understandable, that economic factors that changes cause the interest rate market as a whole to increase or decrease.

4.1.2. PC2: Change in slope

The second PC is a change to the slope of the curve. The long end goes up while the short end goes down or vice versa. This PC usually explains 10%-15% of the curve's historical movement.

4.1.3. PC3: Change in curvature

The third PC is a change to the curvature of the curve. The short and the long end increase while the mid section decrease or vice versa. This PC usually explains 3%-5% of the curve's historical movement.

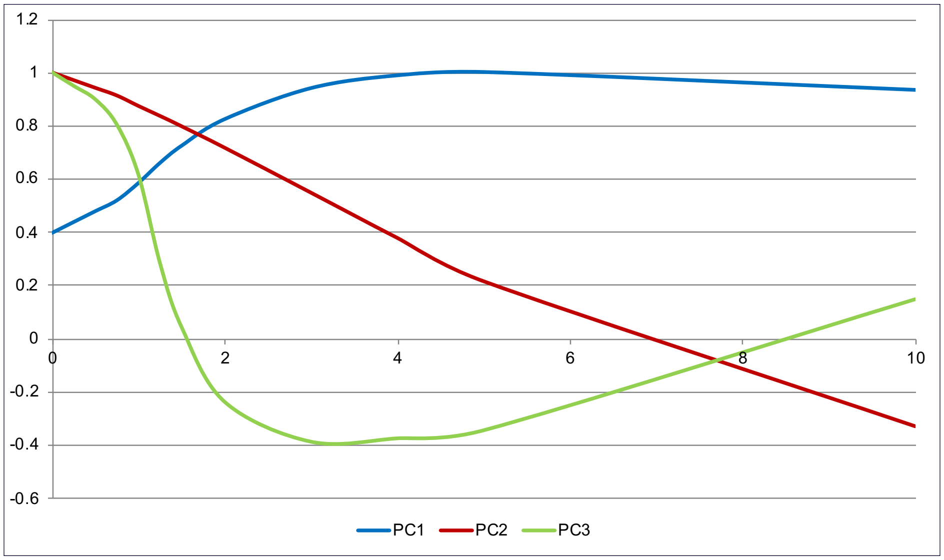


Figure 3. Example of a yield curve's first three principal components.

4.2. Stressing a curve with its principal components

The first three principal components explain the majority of the variance which implies that a linear combination of the principal components can be used to simulate curve changes with high accuracy.

Nasdaq will on a monthly basis evaluate and, if needed, update each yield curve's first three principal components together with a risk parameter that decides how much of this principal component that will be used to simulate the stressed curves. The determination of the risk parameters and other related issues are discussed in the CFM – Calibration document.

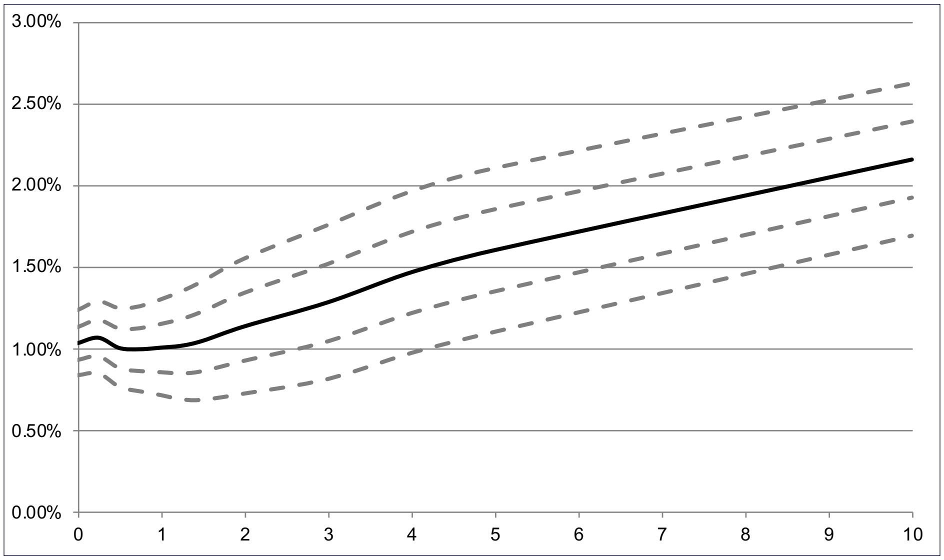


Figure 4. Stressing a curve with its first PC i.e. by different levels of parallel shifts.

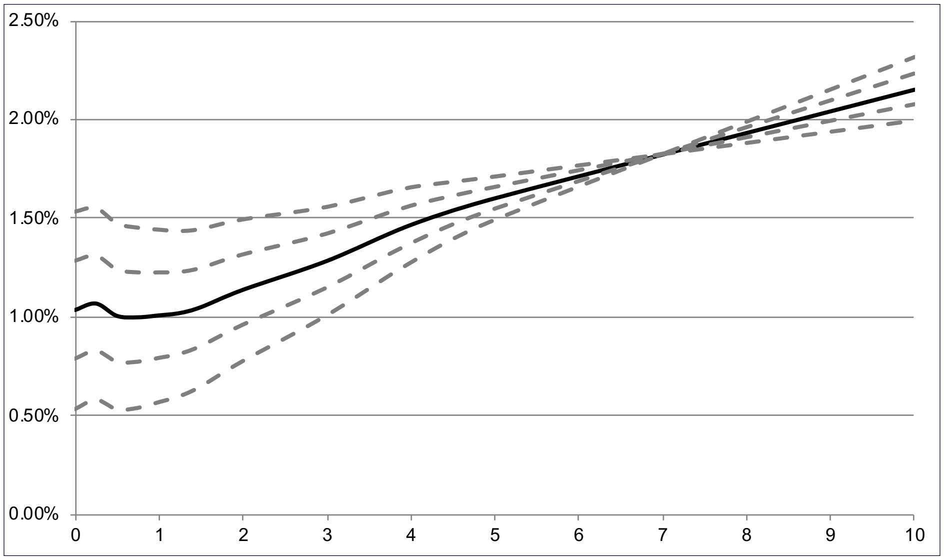


Figure 5. Stressing a curve with its second PC i.e. by different levels of slope changes.

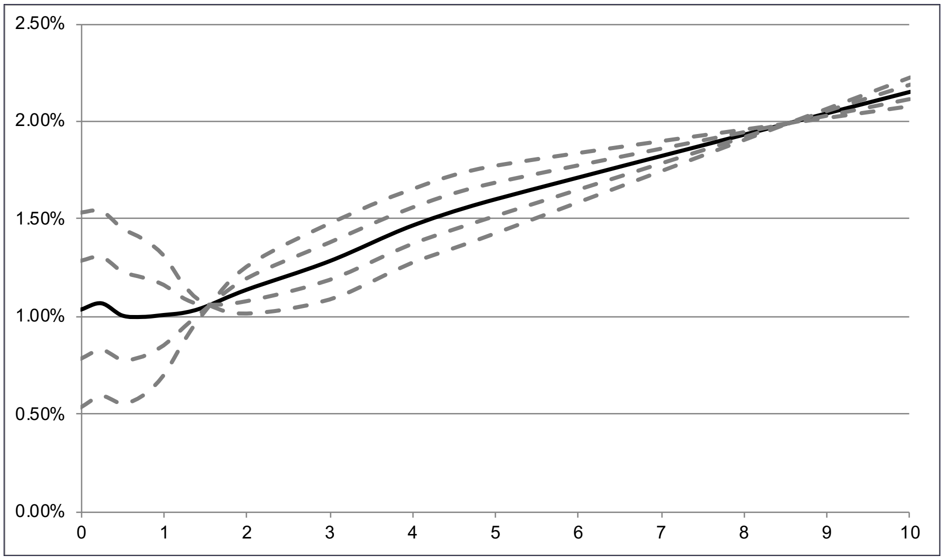


Figure 6. Stressing a curve with its third PC i.e. by different levels of curvature changes.

Nasdaq will on each trading day, t, bootstrap official spot curves (Curve_t). These curves will form the base for all stressed curves. The equation below will be used to simulate the stressed curves from the official curves. Nasdaq defines the yield curves and their principal components as vectors. The margin calculation examples in the end of this document describe this process in more detail.

$$\overline{Curve}_{Stressed} = \overline{Curve}_t + a \cdot \overline{PC1} + b \cdot \overline{PC2} + c \cdot \overline{PC3}$$

a, b and c will range between ± each principal component's risk parameter.

The margin requirement will be determined by the curve scenario which yields the worst market value change.

4.2.1. Conclusion

The concept of Principal Components has been introduced and we have seen examples of how they can be used to produce stressed curves. A more thorough and mathematical description of the principal component analysis can be found in the appendix.

Having covered the key concepts of cash flows, curves and principal components, the scope will now turn toward how they are used in the calculation of the margin requirement for the different fixed income instruments which are cleared at Nasdaq.

5. Margin Calculations

At the core of Nasdaq's CFM methodology are the cash flows of the instruments, the curves that act as price bearers and the principal components of these curves. Exactly how these concepts are used in the calculation of margin requirements differ depending on the market model. This section of the document will in detail explain how the margin requirement is calculated for different instruments.

A common denominator for all instruments is that their initial margin requirement is calculated using a one-curve valuation with a discount factor curve. To have a common methodology for valuation when generating different market scenarios enables the Clearinghouse to offer correlation between different curves.

The instruments differ in how their market values, (and thereby variation margins) are calculated.

5.1. Market models and calculation principles

An overview of the three market models are presented below, together with a classification of how market values and variation margins are calculated. The instruments included in each market model are also listed.

5.1.1. Standardized contracts

All standardized contracts share the trait that their market values are strictly formula based, as defined in their respective contract specifications. No curves are used when calculating the market values of standardized contracts.

Their margin calculations are all done with discount factor curves, and are dependent on how their cash flows are represented.

They differ in how the P/L in the derivatives is treated.

5.1.1.1. Daily Cash Settlement

For interest rate futures, the market value of the contract is settled on a daily basis, and the contracted rate re-written so that the contract's market value is zero. The margin requirement EOD is therefore equal to the initial margin. The margin requirement intraday includes any intraday market value changes.

- **Bond future:** A bond future will be margined against the bond's corresponding yield curve. In the margin calculations, the forward yield of the underlying bond will be calculated using a discount factor curve calibrated on prices of bonds with the relevant issuer.
- **RIBA-future:** For margining purposes, a Riksbank future will be treated as one fixed and one floating cash flow. The floating cash flow will be forecasted using a discount factor curve calibrated using RIBA rates. No discounting of cash flows will take place.
- **NOWA-future:** A NOWA future will be treated as one fixed and one floating cash flow. The floating flow will be forecasted using a discount factor curve calibrated using rates derived from NOWA future prices. No discounting of cash flows will take place.

5.1.1.2. Monthly Cash settlement

For the interest rate forwards offered for clearing at Nasdaq, a monthly settlement of P/L takes place. They are in effect hybrid forwards, where the compounded P/L of the derivative is periodically settled, and the contract rate re-written so that its market value is zero.

- **FRA:** For margining purposes, a forward rate agreement will be treated as one fixed and one floating cash flow. The floating flow will be forecasted using a discount factor curve calibrated using deposit rates, forward rates and swap rates. The same curve will be used for discounting the two cash flows when calculating the contracts NPV.

5.1.1.3. Delivery Only

For interest rate options, no cash settlement of the market value takes place. At termination the underlying instrument, itself either a standardized FRA or bond future, is delivered.

- **Options on FRAs and Bond futures:** Depending on the underlying instrument, the underlying rate/yield will be stressed using curves as described above and then an option pricing formula will be applied to determine the stressed NPV of the option.

5.1.2. Generic Rates Instruments

All the instruments covered within the Clearinghouse's OTC rates clearing offering share the trait of having a valuation which builds on curves. They are all mutually cash collateralized, meaning that the market value of the derivatives is shifted on a daily basis between net debtors and net creditors. An overnight index rate is paid on the accumulated cash collateral, resulting in the need for OIS-discounting and the separation of discounting and forecasting curves.

The Overnight Index Swap uses an instantaneous forward rate curve both for forecasting and discounting. All other instruments are valued using a forward fixing rate curve for forecasting and a discount factor curve for discounting.

- **Interest rate swaps, Tailor Made Forward Rate Agreements and Interest Rate Basis Swaps:** These derivatives will be treated as a series of future cash flows, fixed and floating. For valuation purposes, the floating cash flows will be forecasted with a forward fixing rate curve calibrated using deposit derivatives of the relevant tenor. All cash flows will then be discounted using a discount factor curve calibrated on OIS prices. For margining purposes all forecasting and discounting will be done with the same discount factor curve calibrated on prices of deposits, forward rate agreements and swaps with the relevant floating leg tenor.
- **Overnight Index Swap:** This derivative will be treated as a series of future cash flows, fixed and floating. For valuation purposes, the Overnight Index Swap uses an instantaneous forward rate curve calibrated on OIS prices both for forecasting and discounting. For margining purposes all forecasting and discounting will be done with the same discount factor curve calibrated on OIS prices.

5.1.3. Repo Transactions

For repo transactions, the market valuation and the calculation of margin requirement are performed in the same way. A discount factor curve calibrated on prices of bonds with the relevant issuer is used in both cases. The margin requirement will be the sum of the market value and the initial margin.

- **REPO:** A REPO transaction will be treated as a sold/bought bond spot contract and a corresponding bought/sold bond future contract. These contracts will consist of one cash flow representing the cash payment (the repo consideration) and a series of fixed cash flows representing the bond. For valuation and margining purposes all forecasting and discounting will be done with the same discount factor curve calibrated on prices of bonds with the relevant issuer.

5.2. Detailed Description of Margin Requirement Calculations

5.2.1. Futures

5.2.1.1. Bond futures

NASDAQ offers bond futures on synthetic underlyings with a combination of daily cash settlement and delivery of underlying cash instrument at fixing. The synthetic bond futures contracts have a maturity of two, five or ten years and a fixed annual coupon rate.

It should be noted that the NPV is calculated from a yield curve constructed using prices on cash bonds. The bond futures contracts are not used as calibration instruments and thus the unstressed NPV in margin calculations will slightly deviate from the market value. However, the market value presented in margin reports for the bond futures will be calculated based on the difference between the traded price (r) and today's fixed price (rt).

The synthetic bond future contract is traded on the forward yield of the deliverable bond, but the P/L is calculated using the characteristics of the synthetic bond. Using CFM to calculate margin for these contracts therefore requires some additional steps compared to the usual cash flow forecasting and discounting used for most other interest rate derivatives. In short, the cash flows of the synthetic bond futures will be discounted not by using the yield curve, but by using the forward yield to maturity of the deliverable bond as implied by the yield curve.

Definitions

t	=	Day t
n	=	Outstanding coupons
N	=	Nominal value
Side	=	$\begin{cases} 1 \text{ if the future is bought.} \\ -1 \text{ if the future is sold.} \end{cases}$
C	=	Coupon rate
Q	=	Number of contracts
r_C	=	Contracted yield
r_t	=	Market yield at day t

d	=	Number of days between the contract's settlement date and next coupon payment (30E)
d_e	=	The future contract's expiration date
d_{sett}	=	The future contract's settlement day
d_c	=	Next coupon date
C_i	=	Underlying bond's payment at value date ti
d(0,t)	=	Discount factor at day t.
y	=	Forward price of the deliverable bond, expressed in yield.

Market Valuation

The bond future market valuation is performed by using the following equation to convert the contracted rate, r_c, quoted in yield to a forward price in money.

$$P_{bond}(r_c) = N \cdot \frac{\left(\frac{C}{r} \cdot ((1 + r_c)^n - 1) + 1\right)}{\left((1 + r_c)^{\left(\frac{d}{360} + n - 1\right)}\right)}$$

The same is then done for the market yield, r_t. The profit and loss of the bond future is then calculated as the difference between the contracted future's price, and the current future's price.

$$PnL_{BondFuture} = Side \cdot (P_{bond}(r_t) - P_{bond}(r_c))$$

Margin Requirement

The margin requirement for a bond future is calculated using a discount factor curve calibrated using the market prices of the benchmark bonds of the corresponding issuer. The curve is transformed into a zero coupon spot rate curve and then stressed with three principal components. The curve scenario which yields the worst change in the market value of the contract will determine the margin requirement.

For non-synthetic bonds, the discount factor curve in each stressed curve scenario would be used to calculate the forward value of the underlying bond. The bond future PnL in the margin calculations is then calculated as the difference between the futures price as estimated from the curves, and the contracted futures price as determined from the contracted rate, r.

$$PnL_{BondFuture} = Side \cdot \left(P_{bond}(d(0,t)) - P_{bond}(r)\right)$$

$$P_{bond}(d(0,t)) = \sum_{i=1}^{n-1} d(d_{sett}, t_i) \cdot C + d(d_{sett}, t_n) \cdot (1 + C)$$

For synthetic bonds, the price from the stressed curves is calculated in two steps. First, the futures price of the **deliverable bond** is calculated using the discount function, as described above, and expressed as yield, **y**. This yield is then used in the pricing formula for bonds, using the characteristics of the **synthetic bond,** to get the stressed price of the bond. Finally the profit and loss in each stressed scenario is calculated as the difference between the stressed price and the contracted price.

$$PnL_{BondFuture} = Side \cdot \left(P_{bond}(y) - P_{bond}(r)\right)$$

5.2.1.2. RIBA Future

Contract Description

A RIBA future is a daily cash-settled contract on the Swedish Riksbank's repo rate. The contract base is a fictitious loan extending between two consecutive IMM-dates, and the price is quoted as a compound interest. The final fix of each future is determined by the repo rate between the IMM-date of the contract's end month and the preceding IMM-date. Therefore, during the last three months of a contract's life, the price risk is gradually decreasing as the uncertainty in the final fixing is continuously reduced. CFM will pick up this feature by only regarding the un-fixed part of the closest RIBA future as a floating cash flow. In CFM, the RIBA futures will be margined using a designated RIBA curve, built from the prices on the RIBA contracts.

For further information, consult the contract specifications for the RIBA future.

Definitions

t_m	=	Maturity date of the RIBA i.e. the IMM-date of the contract's end month.
t_{m-1}	=	Preceding IMM-date.
t_{fix}	=	The highest date to which the Riksbank's repo-rate is known. Note that this can be a future date.
Q	=	Quantity.
N	=	Principal amount of the fictive loan.
Side	=	$\begin{cases} 1 \text{ if the RIBA is bought.} \\ 1 \text{ if the RIBA is sold.} \end{cases}$
$r_{i,i+1}$	=	Forward repo rate between ti and ti+1.
$R_{i,i+1}$	=	Historic repo rate between ti and ti+1, expressed as a compounded rate.
r_{first}	=	Estimated rate for the first RIBA contract.
$d_{i,i+1}$	=	Number of days in the floating interest rate period between value dates ti and ti+1 (ACTUAL).
r_c	=	Contracted rate of the RIBA i.e. the last fixing of the RIBA.
r_{c-1}	=	Previously contracted rate of the RIBA i.e. the fixing of the RIBA for the second last business day.
r_m	=	Observed market rate of the RIBA.
$PnL_{RIBA}(r_{i,i+1},r_c)$	=	Profit and loss of a RIBA contract given a forward-forward rate, ri,i+1, and a contracted rate, rc.

Market Valuation

For intraday valuation, the market value of a RIBA future is a function of the contracted rate and the observed market rate.

$$PnL_{RIBA}(r_m, r_c) = Side \cdot Q \cdot N \cdot (r_m - r_c) \cdot \frac{d_{m-1,m}}{360}$$

As the contract is cash settled on a daily basis, the market value is always zero at EOD. The cash settlement amount is calculated as the profit and loss of the contract at EOD, based on the difference between the RIBA fixing and the previous RIBA fixing (or the contracted price for trades executed the same day).

$$PnL_{RIBA}(r_c, r_{c-1}) = Side \cdot Q \cdot N \cdot (r_c - r_{c-1}) \cdot \frac{d_{m-1,m}}{360}$$

Margin Requirement

The margin requirement for the RIBA future is calculated using a discount factor curve calibrated using the market prices of the RIBA futures. The curve is transformed into a zero coupon spot rate curve and then stressed with three principal components. The curve scenario which yields the worst change in the market value of the contract will determine the margin requirement.

In the margin calculations, the discount factor curves are used to forward repo rates as described above. The resulting estimated forward repo rate is then input into the RIBA P/L formula.

$$PnL_{RIBA}(r_{i-1,i}, r_c) = Side \cdot Q \cdot N \cdot (r_{i-1,i} - r_c) \cdot \frac{d_{i-1,i}}{360}$$

When estimating the forward repo rate for the front contract, CFM needs to take into consideration the historic repo rate from the preceding IMM-date up to the highest date to which the Riksbank's repo rate has been determined. This information is used to calculate an estimated rate for the first RIBA contract. Then the P/L can be calculated using this estimated rate.

$$r_{first} = \left(\left(1 + R_{m-1,fix} \cdot \frac{d_{m-1,fix}}{360}\right) \cdot \left(1 + r_{fix,m} \cdot \frac{d_{fix,m}}{360}\right) - 1\right) \cdot \frac{360}{d_{m-1,m}}$$

$$PnL_{RIBA}(r_{first}, r_c) = Side \cdot Q \cdot N \cdot (r_{first} - r_c) \cdot \frac{d_{m-1,m}}{360}$$

As the RIBA is a futures contract, the P/L does not need to be added into a cash flow table in order to calculate the NPV.

5.2.1.3. STIBOR Future

Nasdaq offers clearing of interest rate futures with the 3M Deposits of STIBOR as underlying. These futures are quoted as 100 minus the yield, which means that their P/L dynamics are similar to that of cash bonds. In contrast to the other cleared derivatives with single period forward rates as underlying (RIBA futures and FRAs), an increase in the estimated underlying rate results in a negative P/L. For example, a long position in a STIBOR future can be hedged by a long position in a FRA for the same period. The STIBOR Futures always have an underlying period of 90 days, independently of the actual number of days between the IMM-days.

Definitions

t	=	Trade date.
t_m	=	Maturity date.
Q	=	Quantity.
N	=	Principal amount of the fictive loan.
Side	=	$\begin{cases} 1 \text{ if the future is bought.} \\ -1 \text{ if the future is sold.} \end{cases}$
$r_{i,i+1}$	=	Forward interbank deposit rate between ti and ti+1.
P_c	=	Daily fix
r_c	=	Contracted rate of the STIBOR Future.
r_m	=	Observed market rate of the STIBOR Future.
Row_m	=	Cash flow table value for value date tm.

Market Valuation

The daily fix is quoted in price (100-median value in yield) i.e. (100 - r). The cash flows are decided by the formulas:

$$\boldsymbol{r_c = 100 - P_c}$$

$$\boldsymbol{PnL_{STIBOR}(r_m, r_c) = Side \cdot Q \cdot N \cdot (r_c - r_m) \cdot \frac{90}{360}}$$

As we are here dealing with futures contracts, the P/L doesn't need to be added to any cash flow table for discounting purposes.

Margin Requirement

The margin requirement for a STIBOR future is calculated using a discount factor curve calibrated using the market prices of the corresponding futures. The curve is transformed into a zero coupon rate curve and then stressed with three principal components. The curve scenario which yields the worst change in the market value of the contract will determine the margin requirement.

In the margin calculations, the discount factor curves are used to calculate forward deposit rates. The resulting estimated forward deposit rate is then put into the STIBOR future P/L formula.

$$\boldsymbol{PnL_{STIBOR}(r_{i,i+1}, r_c) = Side \cdot Q \cdot N \cdot (r_c - r_{i,i+1}) \cdot \frac{90}{360}}$$

For the STIBOR futures the P/L does not need to be added into a cash flow table in order to calculate the NPV.

5.2.1.4. NOWA Future

Contract Description

A NOWA future is a daily cash-settled contract which price is based on the NOWA rate governed by the Norwegian Bank. The contract base is a fictitious loan extending between two consecutive IMM-dates, and the price, P, is quoted as 100 minus a compounded interest rate, R

$$P_{NOBA}=100\text{-}R.$$

The final fix of each future is determined by the NOWA rate between the IMM-date of the contract's end month and the preceding IMM-date. Therefore, during the last three months of a contract's life, the price risk is gradually decreasing as the uncertainty in the final fixing is continuously reduced. CFM will pick up this feature by only regarding the un-fixed part of the closest NOWA future as a floating cash flow. In CFM, the NOWA futures will be margined using a designated NOWA curve, built from the prices on the NOWA futures contracts.

Since these futures are quoted as 100 minus the yield the P/L dynamics are similar to that of cash bonds. In contrast to the other cleared derivatives with single period forward rates as underlying (RIBA futures and FRAs), an increase in the estimated underlying rate results in a negative P/L.

In addition, the NOWA futures will have a fixed monetary value, i.e. a fixed tick value in that 1 basis point is always worth 25 NOK. This means that if price is raised by 0.01, the resulting P/L in money will be the same for different NOWA futures, independent upon days between start date and end date. This is in line with international standards for interest rate futures that always have a fixed tick value.

For NOWA futures where the number of days between the start date and end date of the contract is not equal to 90, i.e. not implying a fixed monetary value of 25 NOK given a 360 days a year convention, the fixed monetary value is achieved by using the effective nominal instead of the nominal:

$$Effective\ nominal = PQF \cdot \frac{YD}{d_{i,i+1}} \cdot 100$$

where PQF is the price quotation factor, $d_{i,i+1}$ is the number of days in the floating interest rate period between the start date and end date of the contract and YD is the number of days per year given by the day count convention of the instrument.

In case the number of days between start date and end date would equal 90, then no adjustment of the nominal value is needed.

For further information, consult the contract specifications for the NOWA future.

NOWA publication lag

The fixing prices for the NOWA underlying are published with a publication lag. For NOWA, this means that the logic used for determining fixedDate differs from logic for RIBA. With fixedDate is meant date up to which the rate is considered fixed.

For futures where the underlying fixing is published without publication lag, such as RIBA, the following logic applies for determining fixedDate:

This is the end date of the leg flow for which the following is true:

1. If this is an intra day calculation: leg start date < today's date <= leg end date
2. If this is an end of day calculation: leg start date <= today's date < leg end date

For futures where the underlying fixing is published with publication lag, such as NOWA, the following logic will apply:

This is the maximum end date of any leg flow for the series for which the following is true:

1. If this is an intra day calculation: leg end date < today's date
2. If this is an end of day calculation: leg end date <= today's date

The below table lists fixedDate for RIBA and NOWA on a normal week, i.e. a week without any public holidays:

Day	RIBA	NOWA
Monday intra day	Wednesday this week	Previous Friday
Monday EOD	Wednesday this week	Today
Tuesday intra day	Wednesday this week	Previous day
Tuesday EOD	Wednesday this week	Today
Wednesday intra day	Today	Previous day
Wednesday EOD	Wednesday next week	Today
Thursday intra day	Wednesday next week	Previous day
Thursday EOD	Wednesday next week	Today
Friday intra day	Wednesday next week	Previous day
Friday EOD	Wednesday next week	Today

Definitions

t_m	=	Maturity date of the NOWA i.e. the IMM-date of the contract's end month.
t_{m-1}	=	Preceding IMM-date.
t_{fix}	=	The highest date to which the NOWA rate is known.
Q	=	Quantity.
N	=	Principal amount of the fictive loan.
PQF	=	Price quotation factor.
Side	=	$\begin{cases} 1 \text{ if the NOWA is bought.} \\ -1 \text{ if the NOWA is sold.} \end{cases}$
$r_{i,i+1}$	=	Forward NOWA rate between ti and ti+1.
$R_{i,i+1}$	=	Historic NOWA rate between ti and ti+1, expressed as a compounded rate.

R_{first}	=	Estimated rate for the first NOWA contract.
$d_{i,i+1}$	=	Number of days in the floating interest rate period between value dates ti and ti+1 (ACTUAL).
P_C	=	Contracted price of the NOWA i.e. the last fixing of the NOWA.
P_{C-1}	=	Previously contracted price of the NOWA i.e. the fixing of the NOWA for the second last business day.
P_m	=	Observed market price of the NOWA.
$PnL_{RIBA}(P_{i,i+1},P_C)$	=	Profit and loss of a NOWA contract given a forward-forward rate, $r_{i,i+1}$, and a contracted rate, rc.

Market Valuation

For intraday valuation, the market value of a NOWA future is a function of the contracted price and the observed market price.

$$PnL_{NOWA}(P_m, P_c) = Side \cdot Q \cdot (P_m - P_c) \cdot \frac{d_{m-1,m}}{360} \cdot Effective\ nominal$$

As the contract is cash settled on a daily basis, the market value is always zero at EOD. The cash settlement amount is calculated as the profit and loss of the contract at EOD, based on the difference between the NOWA fixing and the previous NOWA fixing (or the contracted price for trades executed the same day).

$$PnL_{NOWA}(P_c, P_{c-1}) = Side \cdot Q \cdot (P_c - P_{c-1}) \cdot \frac{d_{m-1,m}}{360} \cdot Effective\ nominal$$

Margin Requirement

The margin requirement for the NOWA future is calculated using a discount factor curve calibrated using the market prices of the NOWA futures. The curve is transformed into a zero coupon spot rate curve and then stressed with three principal components. The curve scenario which yields the worst change in the market value of the contract will determine the margin requirement.

In the margin calculations, the discount factor curves are used to calculate forward NOWA rates as described above. The resulting estimated forward NOWA rate is then input into the NOWA P/L formula.

$$PnL_{NOWA}(P_{i-1,i}, P_c) = Side \cdot Q \cdot N \cdot (P_{i-1,i} - P_c) \cdot \frac{d_{i-1,i}}{360} \cdot Effective\ nominal$$

by the relation $P_{i-1,i}=1-R_{i-1,i}$.

When estimating the forward NOWA rate for the front contract, CFM needs to take into consideration the historic NOWA rate from the preceding IMM-date up to the highest date to which the NOWA rate has been determined. This information is used to calculate an estimated rate for the first NOWA contract. Then the P/L can be calculated using this estimated rate.

$$r_{first} = \left(\left(1 + R_{m-1,fix} \cdot \frac{d_{m-1,fix}}{360}\right) \cdot \left(1 + r_{fix,m} \cdot \frac{d_{fix,m}}{360}\right) - 1\right) \cdot \frac{360}{d_{m-1,m}},$$

$$P_{first} = 100 - r_{first},$$

$$PnL_{NOBA}\left(P_{first}, P_c\right) = Side \cdot Q \cdot N \cdot \left(P_{first} - P_c\right) \cdot \frac{d_{m-1,m}}{360} \cdot Effective\ nominal$$

As the NOWA is a futures contract, the P/L does not need to be added into a cash flow table in order to calculate the NPV.

5.2.2. Forwards

5.2.2.1. Forward rate agreements - FRA

A forward rate agreement (FRA) is a forward contract on a fictive forward 3M interbank deposit i.e. a view on the future 3M STIBOR rate. There is no delivery of the underlying loan amount. Only a cash amount corresponding to the interest rate difference between agreed interest rate and the fixing rate will be paid. The buyer of the contract is a fictitious borrower who assumes the obligation to pay the difference between the agreed interest rate and the fixing rate to the seller on condition that the agreed interest rate is higher. If the agreed interest rate is lower than the fixing rate, the buyer is paid the interest rate amount by the seller. The amount is valued three months after the expiration of the FRA contract. However, the actual payment is settled on the settlement day of the FRA contract three months earlier. This means that the valued amount needs to be discounted to the settlement day of the FRA contract.

Equations (17) – (18) are used to insert a FRA into the FRA cash flow table.

Definitions

t	=	Trade date.
t_m	=	Maturity date of the FRA.
Q	=	Quantity.
N	=	Principal amount of the fictive loan.
Side	=	$\begin{cases} 1 \text{ if the FRA is bought.} \\ -1 \text{ if the FRA is sold.} \end{cases}$
$r_{i,i+1}$	=	Forward deposit rate between ti and ti+1.
$d_{i,i+1}$	=	Number of days in the floating interest rate period between value dates ti and ti+1 (30E).
r_c	=	Contracted rate of the FRA.
r_m	=	Observed market rate of the FRA.
$PnL_{FRA}(r_{i,i+1},r_c)$	=	Profit and loss of a FRA contract given a forward-forward rate, ri,i+1, and a contracted rate, rc.
Row_m	=	Cash flow table value for value date tm.

Market Valuation

The market valuation is only dependent on the contracted rate in the position rc, and on the observed market rate, rm.

$$PnL_{FRA}(r_m, r_c) = Side \cdot Q \cdot N \cdot (r_m - r_c) \cdot \frac{d_{i,i+1}}{360}$$

In the market valuation of the FRA contract, no discounting takes place.

Margin Requirement

The margin requirement for a forward rate agreement is calculated using a discount factor curve calibrated using the market prices of the deposits, FRAs and interest rate swaps of the corresponding tenor. The curve is transformed into a zero coupon spot rate curve and then stressed with three principal components. The curve scenario which yields the worst change in the market value of the contract will determine the margin requirement.

For each of the stressed curve scenarios, a forecasted forward deposit rate calculated as follows.

$$r_{i,i+1} = f(0, m_i, m_{i+1}) = \left(\frac{(1 + i(0, m_{i+1}))^{m_{i+1}}}{(1 + i(0, m_i))^{m_i}}\right)^{\frac{1}{m_{i+1} - m_i}} - 1$$

Note that ri,i+1 may need to be converted to the correct day count convention. Using the forecasted deposit rate we can then calculated the profit and loss of the FRA in each curve scenario using the same pricing formula as above.

$$PnL_{FRA}(r_{i,i+1}, r_c) = Side \cdot Q \cdot N \cdot (r_{i,i+1} - r_c) \cdot \frac{d_{i,i+1}}{360}$$

In margin calculations, this profit and loss is then discounted so that a NPV for the FRA is given for each curve scenario.

5.2.3. Interest Rate Options

5.2.3.1. Options on Bond futures

Nasdaq offers clearing of options on the Treasury bond futures. The approach to valuation and margin calculations are the same. First an option price in terms of interest rate basis points is determined. Then this price is converted into money using the BPV of the underlying bond future.

Definitions

Q	=	Quantity of underlying contracts
N	=	Nominal contract size
T	=	Time to expiry
r_S	=	Strike expressed as forward yield
r_m	=	Observed forward yield in the market
r_{est}	=	Forward yield estimated from the curve
σ_r	=	Yield volatility
p	=	Value of option, expressed in yield
BPV	=	Basis point value of underlying bond future

Market Valuation

The observed market price (expressed as a yield) of the underlying bond future, the strike yield, the yield volatility and the time to expiry are entered into the Bachelier option pricing formula. If it is call option, it is entered as a put, and vice versa. This reflects the inverse relation between increases in price and yield. In order to cater for the possibility of negative interest rates, the option price is calculated using Bachelier option price formula. The result is the price of the option expressed in yield. In order to convert this premium to a cash measure, the BPV of the underlying bond future is used.

$$p = Bachelier(r_S, r_m, \sigma_r, T)$$

$$NPV = BPV \cdot p \cdot Q \cdot N$$

Margin Calculation

In a margin calculation setting, the same pricing formulas are used. In each the stressed curve scenarios, an estimated forward yield is used as input in price formula.

$$p = Bachelier(r_S, r_m, \sigma_r, T)$$

5.2.3.2. Options on Forward Rate Agreements

Nasdaq offers clearing of options on FRAs. The approach to valuation and margin calculations are the same. First an option price in terms of interest rate basis points is determined. Then this price is converted into money using the BPV of the underlying bond future.

Definitions

Q	=	Quantity of underlying contracts
N	=	Nominal contract size
T	=	Time to expiry
d_{FRA}	=	Length of underlying FRA, measured in days
r_S	=	Strike expressed as forward deposit rate
r_m	=	Observed forward deposit rate in the market
r_{est}	=	Forward deposit rate estimated from the curve
σ_r	=	Yield volatility
p	=	Value of option, expressed in yield
BPV	=	Basis point value of underlying FRA

Market Valuation

The strike rate, the market rate, the rate volatility and the time to expiry are entered into a Bachelier option pricing formula. The result is the price of the option expressed in yield. In order to convert this premium to a cash measure, the BPV of the underlying FRA is used. For the FRA options a constant BPV per contract, as defined below, is used.

$$p = Bachelier(r_S, r_m, \sigma_r, T)$$

$$BPV = \frac{\frac{d_{FRA}}{360}}{10000}$$

$$NPV = BPV \cdot p \cdot 100 \cdot Q \cdot N$$

Margin Calculation

In a margin calculation setting, the same pricing formulas are used. In each the stressed curve scenarios, an estimated forward yield is used as input in price formula.

$$p = Bachelier(r_S, r_{est}, \sigma_r, T)$$

5.2.4. Generic Rates Instruments

5.2.4.1. Interest rate swaps

Interest rate swaps are derivatives of a specific interbank deposit of a certain tenor, for example swaps with the 3M STIBOR deposit rate as underlying. This means that the floating leg of the swaps will consist of a series of consecutive 3M STIBOR deposits. The fixed leg of these swaps consists of a number of cash flows of equal size. In the valuation of an interest rate swap we therefore deal both with fixed and floating cash flows.

For interest rate swaps different frameworks are used for valuation and margin calculation respectively. Market valuation is done using separate forward and discounting curves. Margin calculations are done with a curve built from the deposit, FRAs and swaps that have the same underlying deposit as the instrument that is being margined.

Definitions

t	=	Trade date.
t_S	=	Start date.
t_e	=	End date.
Q	=	Quantity.

N	=	Principal amount. [For shifting swaps: N could decrease or increase periodically over time]
Side	=	$\begin{cases} 1 \text{ if the swap is sold.} \\ -1 \text{ if the swap is bought.)} \end{cases}$
n_{float}	=	Number of remaining floating cash flows
t_n^{float}	=	Payment date of the n:th floating cash flow
n_{fix}	=	Number of remaining fixed cash flows
t_n^{fix}	=	Payment date of the n:th fixed cash flow
$r_{i,i+1}$	=	Forward-forward swap rate valid from ti to t_{i+1}.
δ_i	=	The day count fraction for a specific interest rate period.
r_{fl}	=	First floating interest rate (this is known at the time when the swap is entered).
R	=	Fixed contracted rate of the interest rate swap. [For shifting swaps: R could decrease or increase periodically over time]
$d_{3M}(0,t)$	=	The discount factor curve constructed from 3M IBOR deposits, FRAs and swaps.
$i_{3M}(0,t)$	=	The zero coupon rate curve corresponding to $d_{3M}(0,t)$.
$d_{OIS}(0,t)$	=	The discount factor curve constructed from O/N deposit and OIS.
F(t)	=	The forward fixing rate curve.

Market Valuation

The market valuation is a done in a dual curve framework. The forecasting of the floating cash flows is done using a forward fixing rate curve, F(t). All cash flows are discounted using a discount factor curve constructed using OIS quotes. The sum of all NPVs gives us the NPV of the SWAP.

$$NPV = Side \cdot N \cdot \left(\sum_{i=1}^{n_{fix}} R \cdot d_{OIS}\left(0, t_i^{fix}\right) - r_{fl} \cdot d_{OIS}\left(0, t_1^{float}\right) - \sum_{j=2}^{n_{float}} F\left(t_{j-1}^{float}\right) \cdot d_{OIS}\left(0, t_j^{float}\right) \right)$$

Margin Calculation

The margin requirement for an interest rate swap is calculated using a discount factor curve constructed using deposits, FRAs and swaps with the same underlying tenor. The curve is transformed into a zero coupon spot rate curve and then stressed with three principal components. The curve scenario which yields the worst change in the market value of the contract will determine the margin requirement.

For each of the stressed curve scenarios, a forecasted forward deposit rate will be calculated for all floating cash flows as follows.

$$r_{i,i+1} = f(0, m_i, m_{i+1}) = \left(\frac{(1 + i(0, m_{i+1}))^{m_{i+1}}}{(1 + i(0, m_i))^{m_i}} \right)^{\frac{1}{m_{i+1} - m_i}} - 1$$

Note that $r_{i,i+1}$ may need to be converted to the correct day count convention.

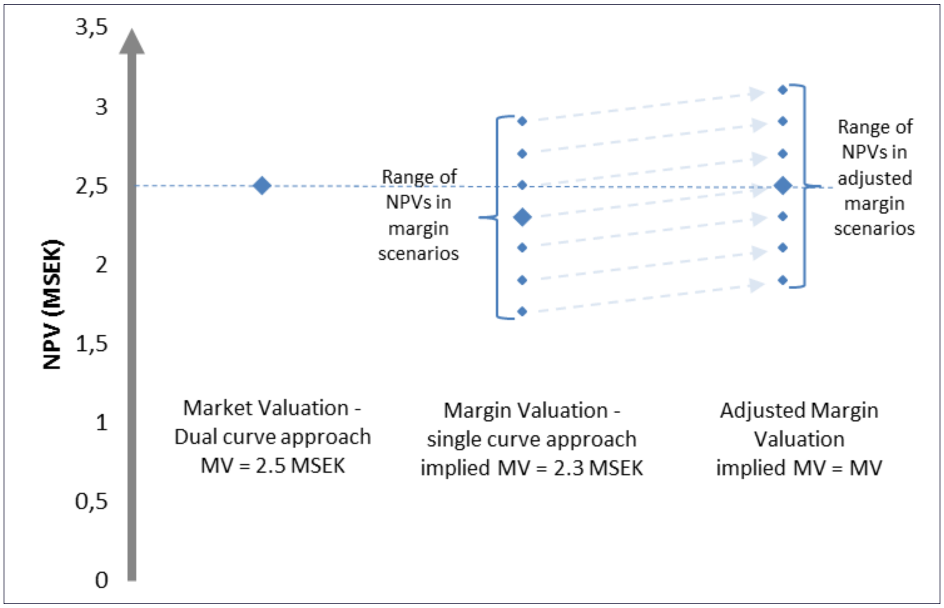
Once we have forecasted all of the floating cash flows, we can calculate the NPV of an interest rate swap in a certain scenario as follows.

$$NPV = Side \cdot N \cdot \left(\sum_{i=1}^{n_{fix}} R \cdot d_{3M}\left(0, t_i^{fix}\right) - r_{fl} \cdot d_{3M}\left(0, t_1^{float}\right) - \sum_{j=2}^{n_{float}} \boldsymbol{r_{j-1,j}} \cdot d_{3M}\left(0, t_j^{float}\right) \right)$$

Market Value adjustment

As different approaches to valuation are used in the market valuation and in the margin calculation, the implied market value from the margin calculations will typically differ from the real market value. Therefore a market value adjustment of the margin calculations is implemented for the generic rate instruments.

By adjusting the NPVs for a certain instrument in the different stressed curve scenarios with the difference between the implied market value and the real market value, it is made sure that the margin scenarios are centered correctly on the true market value.



5.2.4.2. TM FRA

Tailor made FRAs are derivatives of a specific interbank deposit of a certain tenor. In contrast to the standardized IMM-FRAs, the TM FRA can have any start date and end date. Moreover they are also cash collateralized whereas the IMM-FRA are hybrid forwards with monthly settlement of accrued profit and loss.

The principles for the calculation of market value and margin requirements are similar to those for swaps, with the difference that the TM FRA is a much more simple instrument. There is just one floating and one fixed cash flow in a TM FRA.

Definitions

t	=	Trade date.
t_s	=	Start date.
t_e	=	End date.
N	=	Principal amount.
Side	=	$\begin{cases} 1 \text{ if the tm fra is sold.} \\ -1 \text{ if the fra is bought.} \end{cases}$
$r_{s,e}$	=	Forward-forward swap rate valid from ts to te.
δ	=	The day count fraction for the interest rate period
R	=	Fixed contracted rate of the tailor made FRA.
$d_{3M}(0,t)$	=	The discount factor curve constructed from 3M IBOR deposits, FRAs and swaps.
$i_{3M}(0,t)$	=	The zero coupon rate curve corresponding to d3M(0,t).

$d_{OIS}(0,t)$	=	The discount factor curve constructed from the O/N deposit and OIS.
F(t)	=	The forward fixing rate curve.

Market Valuation

The market valuation is a done in a dual curve framework. The forecasting of the floating cash flow is done using a forward fixing rate curve, F(t). Both cash flows are then discounted using a discount factor curve constructed using OIS quotes. The sum of all NPVs gives us the NPV of the SWAP.

$$NPV = Side \cdot N \cdot \delta \cdot d_{OIS}(0, \mathrm{t}_e) \cdot \left(R - F(t_s)\right)$$

Margin Calculation

The margin requirement for a tailor made FRA is calculated using a discount factor curve constructed using deposits, FRAs and swaps with the same underlying tenor. The curve is transformed into a zero coupon spot rate curve and then stressed with three principal components. The curve scenario which yields the worst change in the market value of the contract will determine the margin requirement.

For each of the stressed curve scenarios, a forecasted forward deposit rate will be calculated for the floating cash flow as follows.

$$r_{s,e} = f(0, m_s, m_e) = \left(\frac{(1 + i(0, m_e))^{m_e}}{(1 + i(0, m_s))^{m_s}}\right)^{\frac{1}{m_e - m_s}} - 1$$

Note that $r_{s,e}$ may need to be converted to the correct day count convention.

Once we have forecasted all of the floating cash flows, we can calculate the NPV of a tailor made FRA in a certain scenario as follows.

$$NPV = Side \cdot N \cdot \delta \cdot d_{OIS}(0, \mathrm{t}_e) \cdot \left(R - \mathrm{r}_{s,e}\right)$$

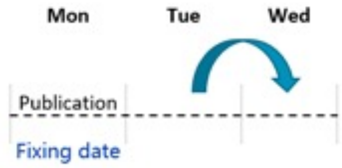
5.2.4.3. OIS

Overnight index swaps are derivatives of the shortest interbank deposit. In the same way as interest rate swaps they have one fixed leg, and one floating leg. However, the floating leg does not generate cash flows in the same frequency as the underlying rate is reset. Instead the O/N or T/N-fixings over one year are integrated into one composite O/N or T/N rate for the whole year, and netted against the yearly fixed cash flows.

The principles for the calculation of market value and margin requirements are similar to those for swaps and TM FRAs, with the difference that the valuation is made with a specifically designed instantaneous forward rate curve.

Publication lag

STIBOR T/N is published the day of its fixing date. The interest rate covers the period from tomorrow until the next day. So, in the example below, the rate enters the Genium INET system on the Monday and is stored as a Fixing Value for the STIBOR instrument with Fixing Date = Monday. Since the rate is Tomorrow/Next, it applies from Tuesday until Wednesday (indicated by the blue arrow).



SWESTR is published the day after its fixing date. The interest rate covers the overnight period from the last business day until the current date. So, in the example below, the rate enters the Genium INET system on the Wednesday and is stored as a Fixing Value for the previous day for the SWESTR instrument with Fixing Date = Tuesday. Since the rate is Overnight, it applies from Tuesday until Wednesday (indicated by the blue arrow).

ESTR is published the day after its fixing date. The interest rate covers the overnight period from the last business day until the current date. So, in the example below, the rate enters the Genium INET system on the Wednesday and is stored as a Fixing Value for the previous day for the ESTR instrument with Fixing Date = Tuesday. Since the rate is Overnight, it applies from Tuesday until Wednesday (indicated by the blue arrow).

Definitions

t	=	Trade date.
t_s	=	Start date.
t_e	=	End date.
N	=	Principal amount.
Side	=	$\begin{cases} 1 \text{ if the swap is sold.} \\ -1 \text{ if the swap is bought.} \end{cases}$
n_{float}	=	Number of remaining floating cash flows.
t_n^{float}	=	Payment date of the n:th floating cash flow.
n_{fix}	=	Number of remaining fixed cash flows.
t_n^{fix}	=	Payment date of the n:th fixed cash flow.
r_{fix_i}	=	The O/N or T/N-fixing at a specific date.
$r_{i,i+1}$	=	Forward-forward composite OIS rate valid from t_i to t_{i+1}.
r_{first}	=	First floating interest rate (partly known, if a T/N Stibor OIS, at the time when the swap is entered).
t_{start}	=	The start date of the first floating rate period.
t_{fix}	=	The end date of the interest rate period to which the last known O/N or T/N-fixing refers.
R	=	Fixed contracted rate of the OIS.
$d_{OIS}(0,t)$	=	The discount factor curve constructed from the T/N or O/N deposit and OIS.
$f_{OIS}(t)$	=	The instantaneous forward rate curve constructed from the T/N or O/N deposit and OIS.

Market Valuation

The market valuation is done using one curve, the instantaneous forward rate curve. The instantaneous forward rate curve is bootstrapped using T/N or O/N deposits (depending on reference rate) and corresponding OIS instruments. This curve is used both for forecasting and for discounting.

The handling of the first floating cash flow requires special attention, as it consists of two parts. For one part the rate is already known, when the first O/N or T/N-fixings are set, and for one part the O/N or T/N-fixings are unknown. We therefore need to calculate the composite average O/N or T/N-rate over the former and forecast the average rate over the remaining period and finally join these two estimates in order to estimate the floating rate for the entire first period.

$$r_{first} = \left(\prod_{t=t_{start}}^{t_{fix}} (1+r_t)^{\frac{\delta}{360}} \cdot e^{\int_{t_{fix}}^{t_1} f_{OIS}(t)} \right) - 1$$

For a given floating cash flow of an OIS, except the first one, the forward compounded OIS rate can be estimated by first calculating the forward discount function over the rate period, $df_{tp \to tq}$, using the following relationship.

$$df_{t_p \to t_q} = \frac{df_{t_q}}{df_{t_p}}$$

The size of the corresponding floating cash flow can then be extracted as:

$$\left(df_{t_p \to t_q}{}^{-1} - 1\right) = \left(\frac{df_{t_q}}{df_{t_p}} - 1\right) = \left(\frac{e^{-\int_0^{t_q} f_{OIS}(t)}}{e^{-\int_0^{t_p} f_{OIS}(t)}} - 1\right) = \left(e^{-\int_{t_p}^{t_q} f_{OIS}(t)} - 1\right)$$

Then the NPV of the OIS can be calculated

$$NPV = Side \cdot N \cdot \left(\sum_{i=1}^{n_{fix}} R \cdot e^{-\int_0^{t_i} f_{OIS}(t)} - r_{first} \cdot e^{-\int_0^{t_1} f_{OIS}(t)} - \sum_{j=2}^{n_{float}} \left(e^{\int_{t_{j-1}}^{t_j} f_{OIS}(t)} - 1 \right) \cdot e^{-\int_0^{t_j} f_{OIS}(t)} \right)$$

Margin Calculation

The margin requirement for an OIS is calculated using a discount factor curve constructed using the shortest deposit, and OIS quotes. For the SWESTR discount factor curve the O/N SWESTR is used as proxy for the shortest deposit since no established T/N market exists.

The curve is transformed into a zero coupon spot rate curve and then stressed with three principal components. The curve scenario which yields the worst change in the market value of the contract will determine the margin requirement.

For each of the stressed curve scenarios, a forecasted forward OIS rate will be calculated for all floating cash flows as follows.

$$r_{i,i+1} = f(0, m_i, m_{i+1}) = \left(\frac{(1 + i(0, m_{i+1}))^{m_{i+1}}}{(1 + i(0, m_i))^{m_i}} \right)^{\frac{1}{m_{i+1} - m_i}} - 1$$

Note that $r_{i,i+1}$ may need to be converted to the correct day count convention.

Once we have forecasted all of the floating cash flows, we can calculate the NPV of an interest rate swap in a certain scenario as follows.

$$NPV = \sum_{i=1}^{n_{fix}} R \cdot d_{OIS}\left(0, t_i^{fix}\right) + r_{fl} \cdot d_{OIS}\left(0, t_1^{float}\right) + \sum_{j=2}^{n_{float}} r_{j-1,j} \cdot d_{OIS}\left(0, t_j^{float}\right)$$

5.2.4.4. REPO

Definitions

Standard	=	Defines if this is a “classic REPO” or a “buy and sell back”.
T	=	Trade date.
t_S	=	Start date.
t_e	=	End date.
t_m	=	Maturity date of the underlying bond.
t_C	=	Date of the underlying bond’s last coupon payment.
$d_{i,j}$	=	Number of days between dates ti and tj (30E).
N	=	Nominal amount of the underlying bond.
C	=	Coupon of the underlying bond.
Side	=	$\begin{cases} \text{1 for a REPO.} \\ \text{-1 for a reversed REPO.} \end{cases}$
r_{repo}	=	Contracted repo rate.
X_S	=	Start consideration.
X_e	=	End consideration.

Market valuation

The market valuation of repos is performed with discount factor curves constructed from the benchmark bonds in the corresponding credit class. Only bonds which build the discount factor curve are allowed as underlying papers in repos, which ensures a correct valuation.

A repo transaction can be de-composed into two bond futures in opposite directions. The cash payments in each of these forwards are referred to as the start and end consideration, and they are both known at the time of the trade. Both of these cash flows are to be regarded as fixed. The bond leg in the two bond futures are represented as the cash flows from the bond, the coupons and the end maturity.

Nasdaq offers clearing of both classic and buy and sell back repos. They differ in the way the end consideration is calculated. In a buy and sell transaction, any coupons that goes to payment during the lifetime of the repo goes to the part who has repoed in the bond. In contrast, in a classic repo transaction, the coupons always stay with the original owner of the bond.

The following equation is used to calculate the end consideration for a classic REPO.

$$X_e = X_s \cdot \left(1 + r_{repo} \cdot \frac{d_{s,e}}{360}\right)$$

The following equation is used to calculate the end consideration for a buy and sell back.

$$X_e = X_s \cdot \left(1 + r_{repo} \cdot \frac{d_{s,e}}{360}\right) - C_i \cdot (1 + r_{repo} \cdot \frac{d_{i,e}}{360})$$

For a repo that hasn’t started yet, the bond cash flows are in different directions and cancel each other out. Therefore the NPV is calculated as

$$NPV = Side \cdot (d_{Bond}(0, t_s) \cdot X_s - \ d_{Bond}(0, t_e) \cdot X_e)$$

For a repo whose start date has passed, the bond exposure is open, and therefore the net present value is calculated taking the cash flows stemming from the bond into consideration.

$$NPV = Side \cdot \left(N \cdot \left(\sum_{i=1}^{n-1} d_{Bond}(0, t_i) \cdot C + d_{Bond}(0, t_n) \cdot (1 + C) \right) - d_{Bond}(0, t_e) \cdot X_e \right)$$

Margin valuation

The margin requirement for the repos is calculated using the same discount factor curve used in the market valuation. The curve is transformed into a zero coupon spot rate curve and then stressed with three principal components. The curve scenario which yields the worst change in the market value of the contract will determine the margin requirement.

6. Correlation of different yield curves

Yield curves in the same currency but with different credit risks can show a historical relationship. A currency's treasury curve may be seen as the base curve, and the other curves in the same currency can be obtained by applying a credit spread to the treasury curve. Nasdaq applies the 3D window method in order to account for correlation of different yield curves in the same currency.

The 3D window method might be difficult to digest for someone that is not used to Nasdaq margin methodology. It is therefore recommended that Appendix III, that describes the 1D window method, is read before this section.

6.1. Sample space: Set of stressed curves

Nasdaq simulates curve changes using the curve's first three principal components. The stressed curves therefore live in a three dimensional sample space[PC1,PC2,PC3].

All principal components will be stressed ± that PC's risk parameter. This implies that all possible curve changes are inside a rectangular prism. This rectangular prism is called the vector cube.

Nasdaq divides the scanning range intervals [-$PC_i \cdot PC_i$'s risk parameter,$PC_i \cdot PC_i$' s risk parameter] into a number of nodes, and the amount of PC_i used in the curve stressing will be evenly distributed over these nodes. Suppose, for example, that the scanning range intervals of the three principal components are divided into 31, 5 and 3 nodes respectively. This would imply that there will be 31 · 5 · 3 = 465 nodes inside the vector cube, and each of these nodes would represent a stressed spot curve.

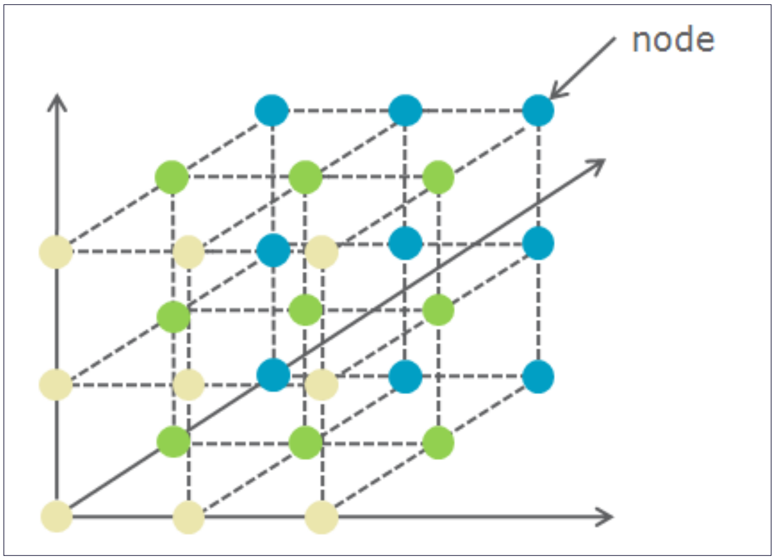


Figure 7. All stressed curves are inside a vector cube.

6.2. Correlation measured per principal components

Two yield curves that are 100% correlated cannot deviate from each other. This implies that their stressing would have to be performed in the same nodes.

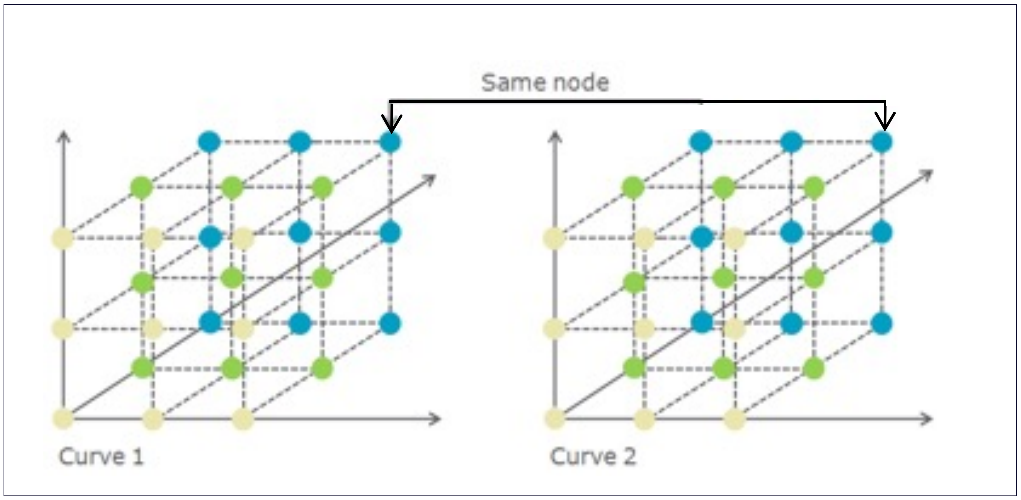


Figure 8. Example on two perfect correlated yield curves.

However, two curves that are not 100% correlated may deviate from each other. When the first curve is stressed in one node, then the other curve may be within one of the neighboring nodes. A volume determines the number of nodes that the two curves may deviate from each other, and the size of this volume is determined by the correlation of the two curves. Nasdaq will determine this size by investigating the yield curves historical correlation in each respective principal component.

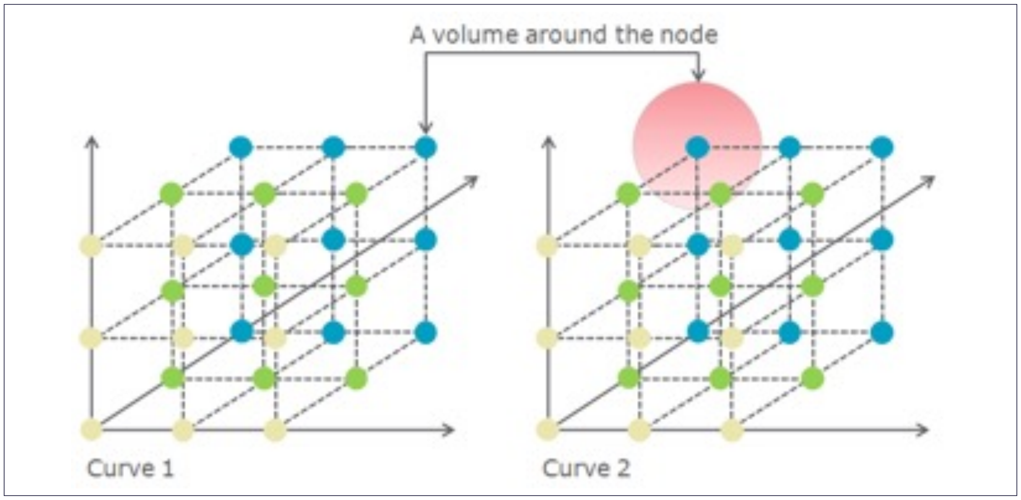


Figure 9. Example of two correlated curves.

6.2.1. PC1

The first principal component is a parallel shift of the entire curve. Suppose there are two curves in the same currency but with different credit rating, for example a treasury curve and a mortgage curve. Further suppose that PC1's risk parameter is 30 basis points for the treasury curve and 33 basis points for the mortgage curve. It would be extremely unlikely that the treasury curve experiences an upward parallel shift of 30 basis points at the same time as the mortgage curve experiences a downward parallel shift of 33 basis points.

Nasdaq defines a window size for PC1. The window size is given as the amount of nodes that two curves in the same window class may deviate in their PC1 stressing. Suppose for example that the treasury and mortgage curves in the above example are put in the same window class and that their PC1 window size is set to 3. This implies that they may maximum deviate 3 nodes from each other in their PC1 stressing. If the treasury curve were to experience an upward parallel shift of 30 basis points (100% of its risk parameter), then that would imply that the mortgage curve can at minimum experience an upward parallel shift of 29 basis points (3 nodes away from the top of its scanning range interval or 87% of its risk parameter).

Table 1. A window size of 3 nodes applied at different nodes.

Treasury		Mortgage		Combined		
Node	**Change**	**Node**	**Change**	**Node**	**Allowed changes (Treasury)**	**Allowed changes (Mortgage)**
1	30	1	33	1	28, 30	31, 33
2	28	2	31	2	26, 28, 30	29, 31, 33
3	26	3	29	3	24, 26, 28	26, 29, 31
4	24	4	26	4	22, 24, 26	24, 26, 29
5	22	5	24	5	20, 22, 24	22, 24, 26
6	20	6	22	6	18, 20, 22	20, 22, 24
7	18	7	20	7	16, 18, 20	18, 20, 22
8	16	8	18	8	14, 16, 18	15, 18, 20
9	14	9	15	9	12, 14, 16	13, 15, 18
10	12	10	13	10	10, 12, 14	11, 13, 15
11	10	11	11	11	8, 10, 12	9, 11, 13
12	8	12	9	12	6, 8, 10	7, 9, 11
13	6	13	7	13	4, 6, 8	4, 7, 9
14	4	14	4	14	2, 4, 6	2, 4, 7
15	2	15	2	15	0, 2, 4	0, 2, 4
16	0	16	0	16	-2, 0, 2	-2, 0, 2
17	-2	17	-2	17	-4, -2, 0	-4, -2, 0
18	-4	18	-4	18	-6, -4, -2	-7, -4, -2
19	-6	19	-7	19	-8, -6, -4	-9, -7, -4
20	-8	20	-9	20	-10, -8, -6	-11, -9, -7
21	-10	21	-11	21	-12, -10, -8	-13, -11, -9
22	-12	22	-13	22	-14, -12, -10	-15, -13, -11
23	-14	23	-15	23	-16, -14, -12	-18, -15, -13
24	-16	24	-18	24	-18, -16, -14	-20, -18, -15
25	-18	25	-20	25	-20, -18, -16	-22, -20, -18
26	-20	26	-22	26	-22, -20, -18	-24, -22, -20
27	-22	27	-24	27	-24, -22, -20	-26, -24, -22
28	-24	28	-26	28	-26, -24, -22	-29, -26, -24
29	-26	29	-29	29	-28, -26, -24	-31, -29, -26
30	-28	30	-31	30	-30, -28, -26	-33,- 31, -30
31	-30	31	-33	31	-30, -28	-33, -31

- A window of 3 nodes applied at node 1 implies that if the treasury curve experiences an upward shift of 28 or 30 basis points, then the mortgage curve may experience an upward shift of 31 or 33 basis points.
- A window of 3 nodes applied at node 10 implies that if the treasury curve experiences an upward shift of 10, 12 or 14 basis points, then the mortgage curve may experience an upward shift of 11, 13 or 15 basis points.
- A window of 3 nodes applied at node 23 implies that if the treasury curve experiences a downward shift of 12, 14 or 16 basis points, then the mortgage curve may experience a downward shift of 13, 15 or 18 basis points.

6.2.2. PC2

The second principal component is a change in the curve's slope. Nasdaq will also define a window size for this principal component. This window size determines the maximum amount of nodes that two curves in the same window class may deviate from each other in terms of their PC2 stressing.

6.2.3. PC3

The third principal component is a change in the curve's curvature. Nasdaq will also define a window size for this principal component. This window size determines the maximum amount of nodes that two curves in the same window class may deviate from each other in terms of their PC3 stressing.

6.3. Window cubes

The window sizes for each principal component constitute a rectangular prism (the window cube) in the [PC1,PC2,PC3] space. This prism determines the number of nodes that the curves in the same window class may deviate from each other.

6.3.1. 3D window method

The 3D window method starts by listing all vector cubes in the same window class next to each other.

- A result vector cube is created and placed next to the other vector cubes.
- A window cube is placed in every top node of the vector cubes.
  - The result vector's value at node i is the sum of each vector cube's lowest net present value from the nodes inside the window cube that is placed at node i.
- The window cubes will slide down all nodes in the vector cubes and the value in the result vector cube will always be the sum of the lowest net present values from the nodes inside the window cubes.

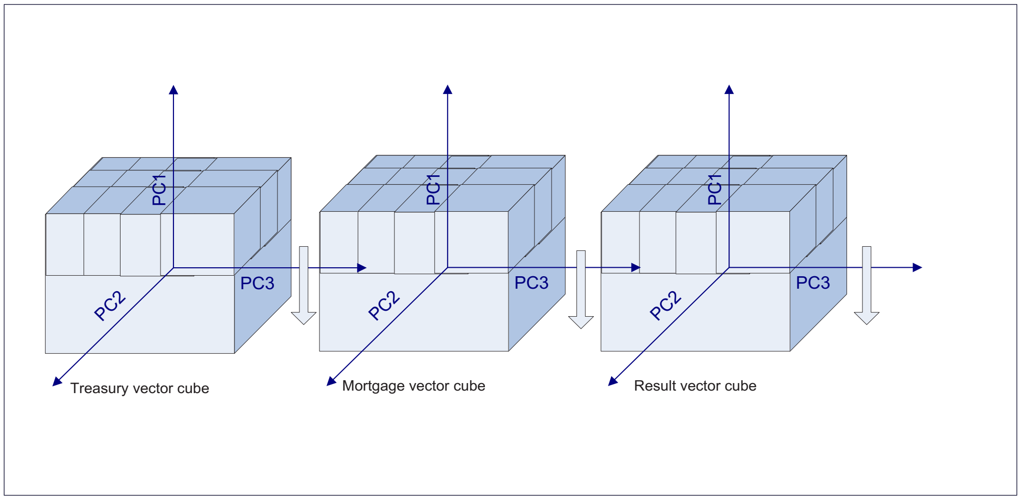


Figure 10. 3D window method applied on the treasury and mortgage vector cubes.

6.3.2. Window trees

The window tree is built up of several layers of window classes and the curves with the closest correlation are placed in the same window class in the bottom of the tree.

The window method is a recursive method; it is first applied to the window classes in the bottom of the window tree. It is here applied on the vector cubes of the cash flow tables within the same window class. During this process a new vector cube, the result vector cube, is created according to the procedures described above. The result vector cube is then combined with result vector cubes from the other window classes in the tree and, as a result, a new result vector cube is created. This procedure is repeated until the top of the window tree has been reached.

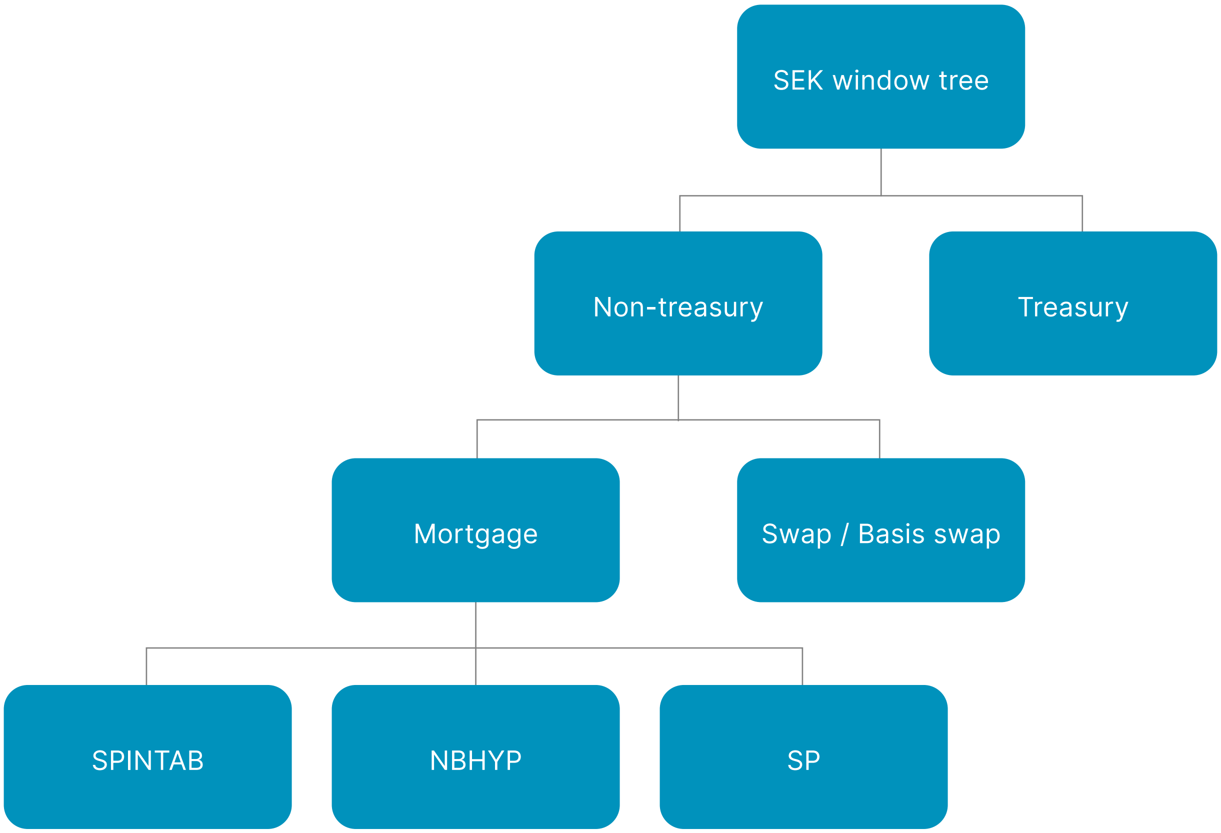


Figure 11. Example of a possible SEK window tree.

7. Appendices

A.1 Appendix I

7.1. Boot Strapping of Yield Curves

The construction of yield curves using the prices and characteristics of selected members in a certain instrument group is referred to as boot strapping. The term comes from the old story of Baron Munchhausen who lifted himself in his owns bootstraps. The analogy is that when constructing a yield curve from bond prices you sometimes need to discount coupons with interest rates on part of the curve which haven't been constructed yet.

Each of the three curves used in CFM has their own construction methodology.

7.1.1. Discount factor curves

Discount factor curves are bootstrapped as a cubic spline. A spline is a mathematical term relating to a set of continuously and smoothly connected polynomials, and the use a cubic version implies that each separate segment of the spline can be described as a cubic polynomial.

$$f = a + b \cdot t + c \cdot t^2 + d \cdot t^3$$

The coefficients a, b, c and d are specifically defined for each segment on the spline. For a specific segment of the spline, t is measured from the start node of the section. Each price bearing instrument that we use in the curve construction defines the end date of a segment through its maturity date. The resulting discount function can therefore be written as:

$$\boldsymbol{d(0,m) = a_{i,i+1} + b_{i,i+1} \cdot (m - m_i) + c_{i,i+1} \cdot (m - m_i)^2 + d_{i,i+1} \cdot (m - m_i)^3}$$

Where $\boldsymbol{m}$ is the maturity for which we want to calculate the discount function and m_i is the start date of the segment of the curve on which $\boldsymbol{m}$ is located.

The problem of boot strapping the curve is to find the coefficients $a_{i,j}, b_{i,j}, c_{i,j}, d_{i,j}$ for each segment of the curve. If these are found then the discount function is defined on the interval $[0, m_{end}]$. The discount function is divided in x segments, were x is the number of calibration instruments. The end node of each segment lay at the end date of the calibration instrument's underlying rate period. We have four unknown coefficients per segment, and this results in a total of 4 · x unknown coefficients.

We solve for the unknown coefficients using a system of linear equations, where we need 4 equations per segment of the curve. We will define these equation using one price equation per calibration instrument and segment, as well as three geometrical equations per connection between segments. The geometric equations ensure that the spline is continuous and smooth. Finally, we define three boundary conditions, determining the start and end characteristics of the spline.

7.1.1.1. Price equations

The discount function must price all calibration instruments correct. The x number of calibration instruments therefore give x price equations. The price equation will look different depending on which type of instrument that is used as calibration instrument.

Bills

The following equation can be used to relate the discount function to the price of a bill.

$$d(0, m_{exp}) = \frac{1}{1 + r_b \cdot \frac{n_{t+2,x}}{360}} \cdot d(0, m_{t+2})$$

where $n_{t+2,x}$ is the actual number of days between t+2 and x, and r_b is the bill’s yield.

Bonds

The following equation can be used to relate the discount function to the dirty price of a bond.

$$d(0, m_{t+3}) \cdot P_{bond} = \sum_i (C_i \cdot d(0, m_i)) + (100 + C_{last}) \cdot d(0, m_{last})$$

Ci is the coupon payment at time T_i.

Deposits

The following equation can be used to relate the discount function to the price of a deposit.

$$d(0, m_{exp}) = \frac{1}{1 + r_d \cdot \frac{n_{t+2,x}}{360}} \cdot d(0, m_{t+2})$$

$n_{t+2,x}$ is the actual number of days between t+2 and x, and r_d is the deposit rate.

Forward rate agreements

The following equation can be used to relate the discount function to the price of a FRA contract.

$$d(0, m_s) \cdot \frac{1}{1 + r_{FRA} \cdot \frac{n_{s,e}}{360}} = d(0, m_e)$$

r_{FRA} is the FRA rate, $n_{s,e}$ is the actual number of days between the start and the end date of the FRA contract’s underlying rate period.

Other derivatives on forward starting deposits, such as RIBA futures and STIBOR futures, also will use this price equation when used in curve generation. Note that for the first RIBA future, the implicit rate is derived through adjusting for the already known repo rate fixing.

Interest rate swaps

The following equation can be used to relate the price of an interest rate swap to the discount function.

$$d(0, m_{t+2}) \cdot N = \sum_i r_f \cdot \frac{n_{i-1,i}}{360} \cdot N \cdot d(0, m_i) + (1 + r_f \cdot \frac{n_{last-1,last}}{360}) \cdot N \cdot d(0, m_{last})$$

N is the principal amount, r_fis the fixed rate of the interest rate swap, and n_(i-1,i) is the number of days between date i-1 and i (measured as 30E).

7.1.1.2. Geometrical equations

Except for the price equations there are also geometrical constrictions to the discount function. It is required $d(0,m),d^{\wedge\prime}(0,m)$ and $d''(0,m)$ are continuous at all nodes. This implies the following relationships (which results in 3x-3 geometrical equations).

Discount function must be continuous

$$a_{i,i+1} + b_{i,i+1} \cdot (m_{i+1} - m_i) + c_{i,i+1} \cdot (m_{i+1} - m_i)^2 + d_{i,i+1} \cdot (m_{i+1} - m_i)^3 - a_{i+1,i+2} = 0$$

Discount function must have a continuous derivative

$$b_{i,i+1} + 2 \cdot c_{i,i+1} \cdot (m_{i+1} - m_i) + 3 \cdot d_{i,i+1} \cdot (m_{i+1} - m_i)^2 - b_{i+1,i+2} = 0$$

Discount function must have a continuous second derivative

$$2 \cdot c_{i,i+1} + 6 \cdot d_{i,i+1}(m_{i+1} - m_i) - 2 \cdot c_{i+1,i+2} = 0$$

7.1.1.3. Boundary conditions

The following three boundary conditions are applied.

The discount function is defined to start at 1 i.e. d(0,0)=1.

$$a_{0,1} = 1$$

The discount function is assumed to have a smooth start i.e. d''(0,0)=1.

$$c_{0,1} = 0$$

The discount function is assumed to reach an equilibrium state i.e. d^'' (0,m_end)=0.

$$2 \cdot c_{end-1,end} + 6 \cdot d_{end-1,end}(m_{end} - m_{end-1}) = 0$$

7.1.1.4. System of linear equations

The price equations, geometrical equations and boundary conditions result in a total of 4 · x equations. These can be solved for all unknown coefficients $a_{i,j}, b_{i,j}, c_{i,j}, d_{i,j}$. When the coefficients have been found it is possible to calculate the discount function for any time to maturity on the interval $[0,m_{end}]$.

7.1.2. Forward Fixing Curves

The forecasting curves are expressed as forward fixing rate curves. This means that every point on the curve represents an estimation of what the forward fixing rate will be at specific future date.

Every forecasting curve describes the future fixings of a specific tenor. As an example, when forecasting a 6M STIBOR fixing, a separate curve will be used as compared to when forecasting a 3M STIBOR fixing.

Nasdaq constructs the STIBOR forecasting curve using piecewise linear interpolation between node points corresponding to the pricing instruments.

The pricing instruments that are included in building the curve should be the deposit of the specific tenor, and derivatives of that deposit. For example, when constructing the 3M STIBOR curve the calibration instruments should be the 3M SEK deposit, the 3M SEK IMM-FRAs and the 3M STIBOR IRS.

Every segment of the curve will be defined by a straight line, expressed in the form $F(t)=a+bt$. For each of these segments, the time t is measured from the node date defining the section's beginning. The number of segments of the curve is equal to the number of FRAs and Swaps used in the curve construction.

The deposit rate will define the start of the curve.

$$a_0 = r_{deposit}$$

A Forward Rate Agreements will have its node at the date of its expiration, let this be t_n for the n:th FRA. Then $a_n = r_{FRA_n}$.

A Swap will have its node at the start date of the last floating cash flow. When incorporating the Swaps into the forecasting curve we need to use the discount factor curve for the instrument group of overnight index swaps, $OIS(t)$. Using the basic property of an at-the-market swap, that the NPV of the floating leg payments is always equal to the NPV of the payments in the fixed leg (C_j), we can define the price equation for the swaps as:

$$\sum \left(F(t_i) \cdot \partial_i \cdot OIS(t_{i+1})\right) = \sum \left(C_j \cdot \partial_j \cdot OIS(t_j)\right)$$

Where ∂_n denotes the day count fraction for a specific rate period n.

In a linear interpolation the equations that guarantee a continuous curve are simply

$$a_i + b_i \cdot t_{i_{end}} = a_{i+1}$$

We then have achieved two equations for each segment on the curve. Those are one price equation per segment, one geometric equation per connection between segments, and one start condition.

7.1.3. OIS Curves

For the purposes of valuing overnight index swaps, a specific OIS curve is used. It is an instantaneous forward rate curve. We build our approach around the fact that we want the continuous forward rate (f_t) to be piecewise linear. Thereby the forward fixing rate will also be almost piecewise linear.

$$f_t = a_i + b_i(t - T_i) \text{ , where } T_i \leq t \leq T_{i+1}$$

We can use all the OIS quotes S_T as calibration prices. Each OIS quote define a discount function between the corresponding period start and end, according to the following equations (∂_T equals the day count fraction expressed in years) :

$S_T = (df_T^{-1} - 1)/\partial_T$ for maturities less than one year

$S_N = (1 - df_N)/\sum_{n=1}^{M} \partial_n \, df_n$ for maturities M over one year

Using these price equations, and the relationship between the discount factor and the continuous rate;

$$df_T = e^{-\int_0^T f_t \, dt}$$

we can then calculate the set of coefficients which define our forward rate curve.

We assume that the first segment of the forward curve is flat, implicating a constant forward rate until T_1. This means $b_0=0$.

The first forward rate is determined by

$$df_{T_1} = e^{-a_0 T_1} \quad \Leftrightarrow \quad a_0 = -\ln\left(df_{T_1}\right)/T_1$$

We thus have determined a_0 and b_0, which define the curve up to the first deposit. Note that for simplicity we measure the time from the spot date, so in the case of T/N Stibor OIS which trade T/N, T=0 is not the current business date, but the following business date. It is easy to convert to discounting to today by multiplying all discount factors with the equivalent of one days discounting.

Determining the rest of the curve involves solving an equation system of two equations and two unknowns for each pricing instrument. One of the equations will express a view on the general shape of the curve, and the other will be derived from the price formulas above.

7.1.3.1. Global curve shape conditions

There are two evident candidates to global curve shape conditions. Either, we may stipulate that the forward rate curve should be continuous at all nodes, then we get:

$$a_{N-1} + b_{N-1} T_{N-1} = a_N$$

Or, we might stipulate that the forward rate may be discontinuous at the nodes, but constant in between nodes. That is:

$$b_N = 0$$

In both cases, there only remains to solve for one coefficient when we look at the pricing formula for a certain instrument.

Which one to choose is a matter of taste, but the last alternative makes the solution less sensible for “overshoot”-effects. The picture below illustrates the differences in between the two methods for 2012-04-18:

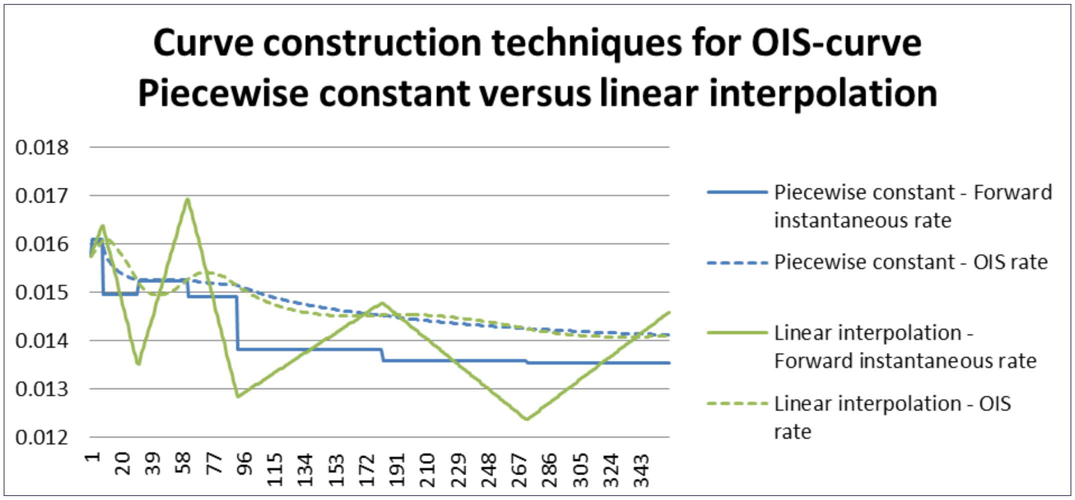


Figure 12. Comparison of different curve construction techniques for the instantaneous forward rate.
A piecewise constant interpolation creates less over-shooting in the implied OIS-rate.

Nasdaq has chosen to use the piecewise constant interpolation technique, but has the possibility to change the interpolation by re-configuring the clearing system.

7.1.3.2. Price Calibration

The second equation we get from the market price S_N and the thereby implied discount function df_N:

$$df_{T_{N+1}} = df_{T_N} e^{-a_N (T_{N+1} - T_N) - b_N \frac{1}{2} (T_{N+1} - T_N)^2}$$

For the pricing instruments with maturities under one year, all of these equations can be solved simultaneously in one equation system, through recognizing that the equation above can be re-written as:

$$\ln \left(\frac{df_{T_{N+1}}}{df_{T_N}}\right) = -a_N (T_{N+1} - T_N) - b_N \frac{1}{2} (T_{N+1} - T_N)^2$$

7.1.3.3. Multi-coupon swaps as price calibration instruments

For the pricing instrument with maturities above one year, the solution finding of b_N will include manipulation that is best done outside of a system of linear equations. For those instruments with any coupon other than the one that occurs at maturity outside of the previously bootstrapped part of the curve (i.e. those instruments with a gap larger than one year to the closest pricing instrument of shorter maturity), some sort of solver has to be applied.

Letting S_{N+1} denote the OIS rate, the price calibration equation for swaps with more than one coupon can be expressed as:

$$S_{N+1} = (1 - df_{N+1}) / \sum_{n=1}^{M} \partial_n \, df_n$$

Which can be rewritten as :

$$S_{N+1} \sum_{n=1}^{M} \partial_n \, df_n = (1 - df_{N+1})$$

Set p and q to the number of coupons which fall on the known and the unknown part of the curve, respectively. That means that the coupons 1 to p lay within the already bootstrapped part of the curve, and the remaining q coupons lay on the part of the curve which remains to be bootstrapped. We can then divide the price equation into a known and an unknown part:

$$S_{N+1} \sum_{n=1}^{p} \partial_n \, df_n + S_{N+1} \sum_{n=M-q+1}^{M} \partial_n \, df_n = (1 - df_{N+1})$$

$$\left(S_{N+1} \sum_{n=1}^{p} \partial_n \, df_n - 1\right) + \left(S_{N+1} \sum_{n=M-q+1}^{M} \partial_n \, df_n + df_{N+1}\right) = 0$$

At this stage of the analysis, the expression within the leftmost parenthesis can be determined, let us denote this known entity A:

$$A = \left(S_{N+1} \sum_{n=1}^{p} \partial_n \, df_n - 1\right)$$

Let us denote the unknown part B, and recognize that it will be a function of a_(N) and b_(N) :a

$$A + B(a_N \, , b_N \,) = 0$$

In the case that q = 1, this equation can be solved analytically:

$$A + \left(S_{N+1} \sum_{n=M-q+1}^{M} \partial_n \, df_n + df_{N+1}\right) = 0$$

$$A + (S_{N+1} \, \partial_M df_M + df_{N+1}) = 0$$

$$\{df_M = df_{N+1}\}$$

$$A + (1 + S_{N+1} \, \partial_M) df_{N+1} = 0$$

$$-df_{N+1} = \frac{A}{(1 + S_{N+1} \, \partial_M)}$$

$$-df_N e^{-a_N \, (T_{N+1} - T_N) - b_N \frac{1}{2} (T_{N+1} - T_N \,)^2} = \frac{A}{(1 + S_{N+1} \, \partial_M)}$$

$$a_N \, (T_{N+1} - T_N) + b_N \frac{1}{2} (T_{N+1} \; - T_N \,)^2 = -\ln\left(\frac{-A}{(1 + S_{N+1} \, \partial_M)} \cdot df_N{}^{-1}\right)$$

Now, either a_N or b_N will be known at this stage, and one can easily solve for the remaining coefficient.

If, on the other hand, q > 1, we will get the following price equation:

$$A + \left(S_{N+1} \sum_{n=M-q+1}^{M} \partial_n \, df_n + df_{N+1} \right) = 0$$

$$\{ df_M = df_{N+1} \}$$

$$A + \left(S_{N+1} \sum_{n=M-q+1}^{M-1} \partial_n \, df_n + (1 + S_{N+1} \partial_M) df_{N+1} \right) = 0$$

$$A + \left(S_{N+1} \sum_{n=M-q+1}^{M-1} \partial_n \cdot df_N \cdot e^{-a_N (T_n - T_N) - b_N \frac{1}{2} (T_n - T_N)^2} + (1 + S_{N+1} \partial_M) \cdot df_N \cdot e^{-a_N (T_{N+1} - T_N) - b_N \frac{1}{2} (T_{N+1} - T_N)^2} \right) = 0$$

At this stage either a_(N)or b_N will be known and the equation above will only contain one unknown variable. However, since we are looking at a sum of power functions we cannot solve this problem analytically. Therefore, a solver algorithm is applied at this stage.

A.2 Appendix II

7.2. Principal components analysis

Nasdaq will stress each yield curve with its first three principal components. The principal component analysis will be performed outside of the GENIUM INET system and the principal components together with their stress levels will be entered into GENIUM INET as risk parameters. This appendix describes the principal component analysis.

7.2.1. Objective

The objective is to find independent (uncorrelated) moves of a yield curve; these will later be used to simulate changes to the yield curve.

7.2.2. Input data

Nasdaq defines the yield curves as interest rate values at different times to maturities (nodes).

Time to maturity	m0	…	mi	…	mend
Spot rate	$i(t,m0)$	…	$i(t,mi)$	…	$i(t,mend)$

The input data to the principal components analysis is a time series of historical changes to these node values.

Date	m0	…	mend
ti	$\lvert i(t_i,m_0)-i(t_{i-1},m_0)\rvert$	…	$\lvert i(t_i,m_{end})-i(t_{i-1},m_{end})\rvert$
ti-1	$\lvert i(t_{i-1},m_0)-i(t_{i-2},m_0)\rvert$	…	$\lvert i(t_{i-1},m_{end})-i(t_{i-2},m_{end})\rvert$
…	…	…	…
ti-500	$\lvert i(t_{i-500},m_0)$ $-i(t_{i-501},m_0)\rvert$	…	$\lvert i(t_{i-500},m_{end})-i(t_{i-501},m_{end})\rvert$

7.2.3. Definitions

7.2.3.1. Covariance matrix

The covariance matrix contains information on each node's historical variance as well as the covariance between the different nodes. The covariance matrix is defined accordingly.

$$COV(i,j) = \frac{\sum(x_i - \bar{x}_i)(x_j - \bar{x}_j)}{n}$$

x_i is the daily change of node i, x_j is the daily change of node j, and n is the total number of observations.

When performing the principal components analysis it is important that the input data is arranged so that the means $\bar{x}_i$ and $\bar{x}_j$ are zero[1]. If this is not correct then the means must be removed from the time series before the analysis proceeds.

7.2.3.2. Principal components

The principal components are defined as the eigenvectors, λ, to the covariance matrix.

$$\overleftrightarrow{COV} \cdot \bar{\lambda} = \sigma \cdot \bar{\lambda}$$

The eigenvectors are orthogonal i.e. independent (uncorrelated). An eigenvector's eigenvalue, σ, reveals how much of the curve's total variance that is explained by this eigenvector. It should be noted that this definition implies that the principal components are in fact lists of changes to the interest rate values at the nodes.

Time to maturity	m_0	...	m_i	...	m_{end}
Spot rate change	1	...	0,9	...	0,4

7.2.3.3. Problem

The first problem is to find the eigenvalues, σ, to the covariance matrix. This is done by solving Equation (35). I in Equation (35) stands for the identification matrix.

$$det\left(\overleftrightarrow{COV} - \sigma \cdot \overleftrightarrow{I}\right) = 0$$

When the eigenvalues are found, then each of them can be inserted into Equation (44), resulting in a system of linear equations that can be solved for their corresponding eigenvectors, λ.

Solution

If the yield curve is defined on x nodes, then the covariance matrix will have size x • x. Equation (45) will then result in x eigenvalues (assuming no doublets) and hence also x eigenvectors/principal components.

If the size of all eigenvalues is compared, then it is possible to determine the importance of each principal component. The table below shows the relative importance of the SEK swap curve's first seven principal components.

Principal component	PC1	PC2	PC3	PC4	PC5	PC6	PC7
Explanation factor	76,5%	12,7%	5,1%	2,0%	0,9%	0,7%	0,4%

1 Please see Jolliffe, I.T. Principal Components Analysis, 2nd edition, Springer series in statistics

The Risk parameters are dependent on the principal components. From the principal components analysis the eigenvectors are by definition orthonormal i.e. orthogonal vectors with magnitude 1. Nasdaq has chosen to scale the eigenvectors/principal components and that will affect both the magnitude of the eigenvector and the risk parameter i.e. the greater the magnitude the lower the risk parameter. However, the direction of the eigenvector will not change which means that the actual stress (Risk parameter * $\overline{PC}$) will not change depending of the magnitude of the principal components

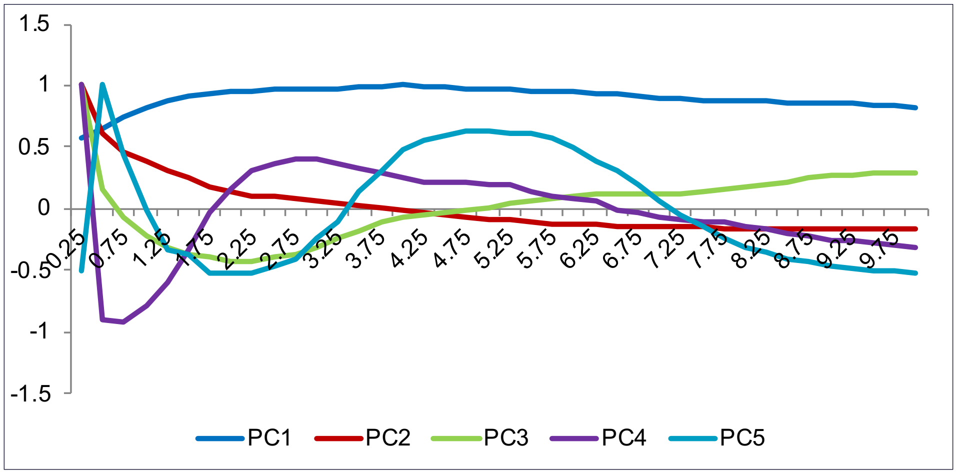


Figure 13. First five scaled principal components of the SEK swap curve.

A.3 Appendix III

7.3. One-dimensional window method

This appendix gives an example on the one-dimensional window method. The example is fictive (the one dimensional window method will never be applied in Nasdaq CFM), but reading the example will hopefully facilitate the understanding of the multi-dimensional window methods used in Nasdaq CFM.

7.3.1. Positions

The position in this example is a EURUSD basis swap. For a given set of yield curves the stressed net present value is given in the cash flow table below.

Value date	USD	EUR
...	...	...
NPV stressed	**1 000 000**	**-667 315**

These stressed net present values must be converted into the margin base currency (SEK). If the USDSEK or EURSEK spot exchange rates changes, then the converted value will also change. This risk is accounted for by stressing the spot exchange rates.

7.3.2. Margin data

The margin data is given in the table below.

Currency pair	Spot rate	Risk parameter
USDSEK	6,86	4%
EURSEK	10,28	3%

7.3.3. Scanning range intervals

Nasdaq stresses the spot exchange rates upwards and downwards with the appropriate risk parameters. This results in the following two scanning range intervals.

$$\mathit{Scanning\ Range}_{USDSEK} = [\,6{,}86 \cdot (1 - 0{,}04), 6{,}86 \cdot (1 + 0{,}04)\,] = [\,6{,}59, 7{,}13\,]$$

$$\mathit{Scanning\ Range}_{EURSEK} = [\,10{,}28 \cdot (1 - 0{,}03), 10{,}28 \cdot (1 + 0{,}03)\,] = [\,9{,}97, 0{,}59\,]$$

7.3.4. Vector files

Nasdaq produces a USD and a EUR vector file. The vector files contain 31 nodes and the USDSEK and EURSEK spot exchange rates will be varied, evenly distributed over their scanning range intervals, in these nodes. Every node further contains the stressed net present value converted into SEK using the node's spot exchange rate.

7.3.5. Window class

In this example it is supposed that the USDSEK and EURSEK currency pairs are in the same window class and that this window class has a window size of 11 nodes.

7.3.6. ne-dimensional window method

The one-dimensional window method starts by listing the USD and EUR vector files next to each other and by creating a "USD, EUR result vector" (refer to the figure below).

7.3.6.1. indow starts in node 1

A window of 11 nodes is then placed in the top node of the vector files. The window represent the maximum amount that the USDSEK and EURSEK spot exchange rates are anticipated to deviate from each other. The value at node 1 of the result vector is the sum of the worst outcomes from the nodes in the USD and EUR vector files that lay inside of the window.

In this example this is node 6 of the USD vector file (i.e. that the USDSEK spot rate goes up to 7,04) and node 1 of the EUR vector file (i.e. that EURSEK spot rate goes up to 10,59). This combined result is entered into node 1 of the result vector.

USD vector file			EUR vector file			USD, EUR result vector			
Node	**USDSEK**	**NPV (SEK)**	**Node**	**EURSEK**	**NPV (SEK)**	**Node**	**USDSEK**	**EURSEK**	**NPV (SEK)**
1	7,13	7 134 400	**1**	**10,59**	**-7 065 800**	**1**	**7,04**	**10,59**	**-22 867**
2	7,12	7 116 107	2	10,57	-7 052 080	2			
3	7,10	7 097 813	3	10,55	-7 038 360	3			
4	7,08	7 079 520	4	10,53	-7 024 640	4			
5	7,06	7 061 227	5	10,51	-7 010 920	5			
6	**7,04**	**7 042 933**	6	10,49	-6 997 200	6			
7	7,02	7 024 640	7	10,47	-6 983 480	7			
8	7,01	7 006 347	8	10,44	-6 969 760	8			
9	6,99	6 988 053	9	10,42	-6 956 040	9			
10	6,97	6 969 760	10	10,40	-6 942 320	10			
11	6,95	6 951 467	11	10,38	-6 928 600	11			
12	6,93	6 933 173	12	10,36	-6 914 880	12			
13	6,91	6 914 880	13	10,34	-6 901 160	13			
14	6,90	6 896 587	14	10,32	-6 887 440	14			
15	6,88	6 878 293	15	10,30	-6 873 720	15			
16	6,86	6 860 000	16	10,28	-6 860 000	16			

USD vector file			EUR vector file			USD, EUR result vector			
Node	**USDSEK**	**NPV (SEK)**	**Node**	**EURSEK**	**NPV (SEK)**	**Node**	**USDSEK**	**EURSEK**	**NPV (SEK)**
17	6,84	6 841 707	17	10,26	-6 846 280	17			
18	6,82	6 823 413	18	10,24	-6 832 560	18			
19	6,81	6 805 120	19	10,22	-6 818 840	19			
20	6,79	6 786 827	20	10,20	-6 805 120	20			
21	6,77	6 768 533	21	10,18	-6 791 400	21			
22	6,75	6 750 240	22	10,16	-6 777 680	22			
23	6,73	6 731 947	23	10,14	-6 763 960	23			
24	6,71	6 713 653	24	10,12	-6 750 240	24			
25	6,70	6 695 360	25	10,09	-6 736 520	25			
26	6,68	6 677 067	26	10,07	-6 722 800	26			
27	6,66	6 658 773	27	10,05	-6 709 080	27			
28	6,64	6 640 480	28	10,03	-6 695 360	28			
29	6,62	6 622 187	29	10,01	-6 681 640	29			
30	6,60	6 603 893	30	9,99	-6 667 920	30			
31	6,59	6 585 600	31	9,97	-6 654 200	31			

Figure 14. A window starts at the top node of the vector files. The value in the result vector is the sum of the worst outcome within the window.

7.3.6.2. Window slides down

The window will slide down all 31 nodes of the vector files. The value in the result vector is always the sum of the worst outcomes within the window.

At note 16 this is equal the value from node 21 of the USD vector file (i.e. that the USDSEK spot rate goes down to 6,77) plus the value from node 11 of the EUR vector file (i.e. that the EURSEK spot rate goes up to 10,38).

USD vector file			EUR vector file			USD, EUR result vector			
Node	**USDSEK**	**NPV (SEK)**	**Node**	**EURSEK**	**NPV (SEK)**	**Node**	**USDSEK**	**EURSEK**	**NPV (SEK)**
1	7,13	7 134 400	1	10,59	-7 065 800	1	7,04	10,59	-22 867
2	7,12	7 116 107	2	10,57	-7 052 080	2	7,02	10,59	-41 160
3	7,10	7 097 813	3	10,55	-7 038 360	3	7,01	10,59	-59 453
4	7,08	7 079 520	4	10,53	-7 024 640	4	6,99	10,59	-77 747
5	7,06	7 061 227	5	10,51	-7 010 920	5	6,97	10,59	-96 040
6	7,04	7 042 933	6	10,49	-6 997 200	6	6,95	10,59	-114 333
7	7,02	7 024 640	7	10,47	-6 983 480	7	6,93	10,57	-118 907
8	7,01	7 006 347	8	10,44	-6 969 760	8	6,91	10,55	-123 480
9	6,99	6 988 053	9	10,42	-6 956 040	9	6,90	10,53	-128 053
10	6,97	6 969 760	10	10,40	-6 942 320	10	6,88	10,51	-132 627
11	6,95	6 951 467	**11**	**10,38**	**-6 928 600**	11	6,86	10,49	-137 200
12	6,93	6 933 173	12	10,36	-6 914 880	12	6,84	10,47	-141 773
13	6,91	6 914 880	13	10,34	-6 901 160	13	6,82	10,44	-146 347
14	6,90	6 896 587	14	10,32	-6 887 440	14	6,81	10,42	-150 920

USD vector file			EUR vector file			USD, EUR result vector			
Node	**USDSEK**	**NPV (SEK)**	**Node**	**EURSEK**	**NPV (SEK)**	**Node**	**USDSEK**	**EURSEK**	**NPV (SEK)**
15	6,88	6 878 293	15	10,30	-6 873 720	15	6,79	10,40	-155 493
16	6,86	6 860 000	16	10,28	-6 860 000	**16**	**6,77**	**10,38**	**-160 067**
17	6,84	6 841 707	17	10,26	-6 846 280				
18	6,82	6 823 413	18	10,24	-6 832 560				
19	6,81	6 805 120	19	10,22	-6 818 840				
20	6,79	6 786 827	20	10,20	-6 805 120				
21	**6,77**	**6 768 533**	**21**	**10,18**	**-6 791 400**				
22	6,75	6 750 240	22	10,16	-6 777 680				
23	6,73	6 731 947	23	10,14	-6 763 960				
24	6,71	6 713 653	24	10,12	-6 750 240				
25	6,70	6 695 360	25	10,09	-6 736 520				
26	6,68	6 677 067	26	10,07	-6 722 800				
27	6,66	6 658 773	27	10,05	-6 709 080				
28	6,64	6 640 480	28	10,03	-6 695 360				
29	6,62	6 622 187	29	10,01	-6 681 640				
30	6,60	6 603 893	30	9,99	-6 667 920				
31	6,59	6 585 600	31	9,97	-6 654 200				

Figure 15. The window will slide down all 31 nodes of the vector files.

7.3.6.3. Margin requirement

When the window has slide down all nodes of the vector files, then the USD, EUR result vector is filled with values. The margin requirement for the combined position is the worst outcome in the USD, EUR result vector.

In this example this is equal to SEK -205 800. This value is taken from node 26 of the USD, EUR result vector and it corresponds to the scenario were the USDSEK spot rate goes down to 6,59 and the EURSEK spot rate goes down to 10,18.

It should be noted that a margin requirement of SEK -205 800 is approximately 43% compared to the margin requirement given if the net present values had been converted independent of each other.

Node	USDSEK	EURSEK	NPV (SEK)
1	7,04	10,59	-22 867
2	7,02	10,59	-41 160
3	7,01	10,59	-59 453
4	6,99	10,59	-77 747
5	6,97	10,59	-96 040
6	6,95	10,59	-114 333
7	6,93	10,57	-118 907
8	6,91	10,55	-123 480
9	6,90	10,53	-128 053
10	6,88	10,51	-132 627
11	6,86	10,49	-137 200
12	6,84	10,47	-141 773

Node	USDSEK	EURSEK	NPV (SEK)
13	6,82	10,44	-146 347
14	6,81	10,42	-150 920
15	6,79	10,40	-155 493
16	6,77	10,38	-160 067
17	6,75	10,36	-164 640
18	6,73	10,34	-169 213
19	6,68	10,32	-173 787
20	6,66	10,30	-178 360
21	6,64	10,28	-182 933
22	6,62	10,26	-187 507
23	6,60	10,24	-192 080
24	6,59	10,22	-196 653
25	6,59	10,20	-201 227
26	6,59	10,18	-205 800
27	6,59	10,16	-192 080
28	6,59	10,14	-178 360
29	6,59	10,12	-164 640
30	6,59	10,09	-150 920
31	6,59	10,07	-137 200

Figure 16. The margin requirement is the worst outcome within the USD, EUR result vector.